



# Analysis of momentum distributions of projectile fragmentation products

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In the Goldhaber model\* of the projectile fragmentation, the removal of independent nucleons from the projectile results in a Gaussian momentum distribution. The width of this distribution is given by the expression:

$$\sigma_{\parallel} = \sigma_0 \sqrt{\frac{A_F (A_P - A_F)}{A_P - 1}}$$

where  $A_F$  is the fragment mass,  $A_P$  is the projectile mass, and  $\sigma_0$  is the reduced width related to the Fermi momentum.

$$\sigma_0 = p_F / \sqrt{5} \approx 90 \text{ MeV} / c$$

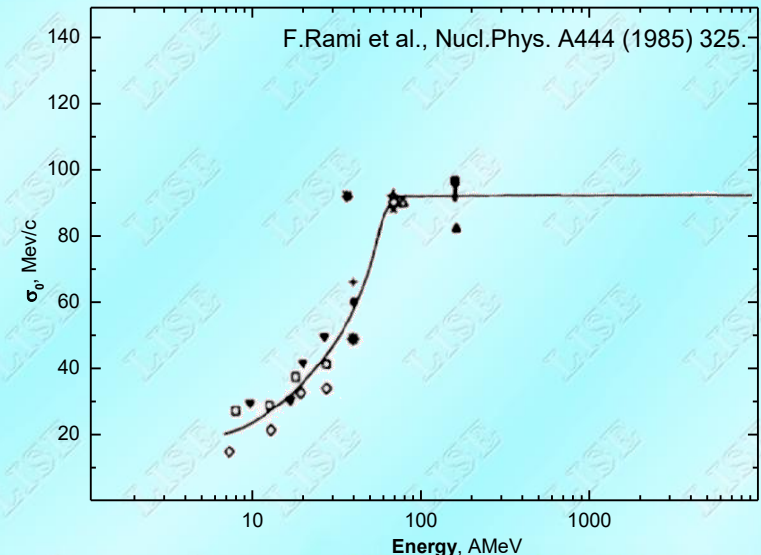
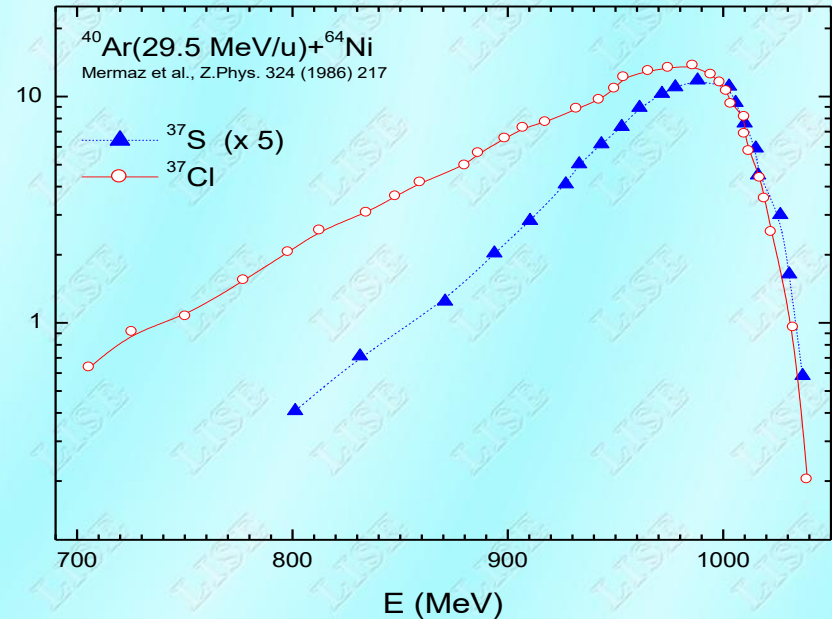
The fragment velocity is supposed to be equal to the projectile velocity.

\* A.S.Goldhaber, Phys.Lett. **B53** (1974) 306.

# Discrepancy with experimental results

However, Goldhaber's model is unable to take into account the following things observed at low energies:

- the differences in widths associated with nuclides of the same mass
- the occurrence of an exponential tail in momentum distributions in reactions
- the reduction of the fragment to projectile velocity ratio ( $v/v_0$ )
- the anomalously small values of reduced width  $\sigma_0$



# Different models : fragment velocity

Further different models were developed to explain these phenomena (both theoretical, and empirical parameterizations). Some models are entered in the LISE++ code\* :

[1] V.Borrel et al., Z.Phys.A 314 (1983) 191. (solid curve)

removing 8 MeV per ablated nucleon:

$$\frac{v_F}{v_P} = \sqrt{1 - \frac{B_n(A_P - A_F)}{A_F E_P}}$$

[2] F.Rami et al., Nucl.Phys. A444 (1985) 325.

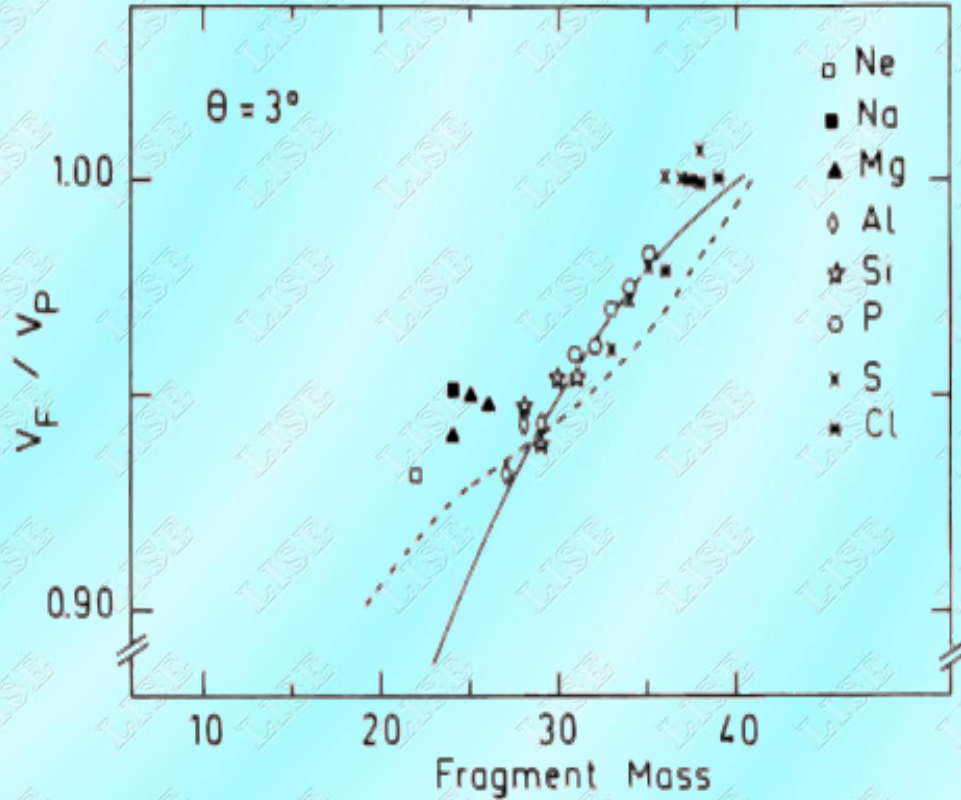
(dotted curve)

Fragment velocity on the basis of surface energy excess:

$$\frac{v_F}{v_P} = \sqrt{1 - \frac{2S}{A_F E_P}}$$

where  $S$  is the surface energy of contact and equal to  $2\gamma s$ .  
 $\gamma$  – the nuclear surface tension coefficient ( $0.95 \text{ MeV/fm}^2$ ),  
 $s$  – the area of the interface between the abraded zone and the remaining fragment.

[3] D.J.Morrissey, Phys.Rev. C39 (1989) 460.



Ratio of the fragment to projectile velocities versus the mass of the fragment in the  $^{40}\text{Ar}(26.5\text{A MeV}) + ^{68}\text{Zn}$  reaction [2].

\* See the poster “LISE++ : design your own spectrometer “

# Different models : Longitudinal momentum distribution widths

## 1. A.S.Goldhaber, Phys.Lett. B53 (1974) 306

$$\sigma_{||} = \sigma_0 \sqrt{\frac{A_F (A_P - A_F)}{A_F - 1}},$$

where  $\sigma_0 = 90 \text{ MeV}/c$ .

## 2. D.J.Morrissey, Phys.Rev. C39 (1989) 460

$$\sigma_{||} = \frac{150}{\sqrt{3}} \sqrt{A_P - A_F}$$

### “Coulomb correction”

This correction becomes significant only for energies below 20 MeV/u.

$$\sigma_0^* = \sigma_0 (1 - E_B / E_{CM})^{1/2}$$

## 3. W.A.Friedman, Phys.Rev. C27(1983) 569.

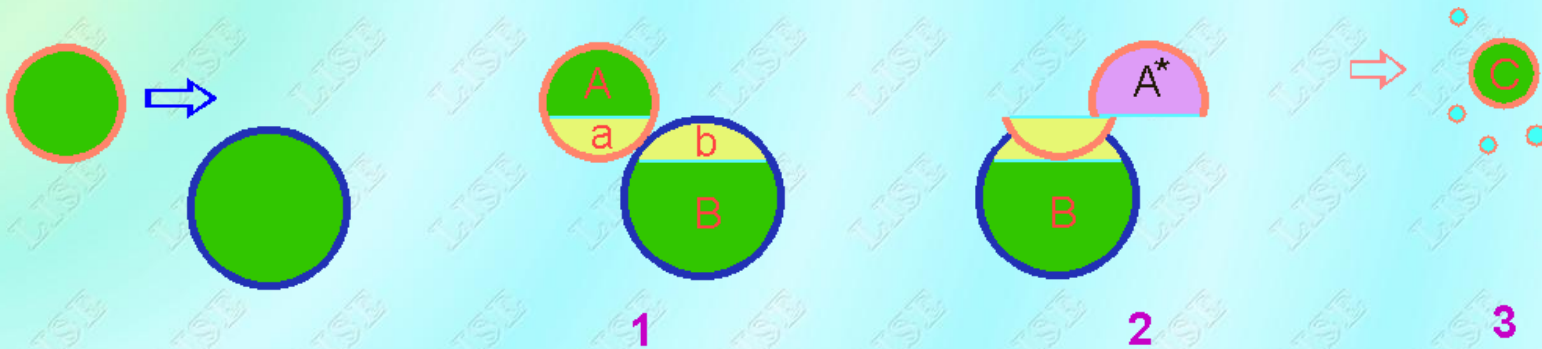
The model relates the widths of distributions to the separation energies and an absorptive cutoff radius. In this work a wave function  $\psi_{F-R}(r)$  is entered which describes the relative separation between the observed fragment (F) and the removed portion of the projectile (R) to calculate only the region outside the absorption:

$$\psi_{F-R}(r) \cong e^{-\mu r} / r, \quad \mu = \sqrt{2m_r E_s}$$

where  $m_r$  is the reduced mass and  $E_s$  is the separation energy. Including the lowest order of the Coulomb potential on the bound state wave function tail, the resulting distribution has gaussian form with width:

$$\sigma_{||}^2 = \frac{\mu}{2x_0} \left[ \frac{1 + 0.5y}{\sqrt{1 + y}} + \frac{1}{\mu x_0} \right], \quad y = Z_1 Z_2 / x_0 E_s$$

# 3-step projectile fragmentation model



Process

Momentum distribution

## 1 Abrasion

Removal of the part "a"  
(statistics)

Gaussian

$$\psi(p_{pf}) = \frac{1}{\sqrt{2\pi} \sigma_{pf}} \exp\left(-\frac{(p_{pf} - p_{0pf})^2}{2\sigma_{pf}^2}\right)$$

## 2 Friction - loss of kinetic energy

Transformation into the  
internal degrees of freedom.  
Exchange of nucleons

Exponential  
attenuation

$$\phi(p_1, p_2) = \frac{1}{\tau} \exp\left(-\frac{p_2 - p_1}{\tau}\right)$$

## 3 Ablation

light nuclei emission,  
gamma-emission

Broadening

velocity peak maximum  
does not shift

# Convolution model: “Universal parameterization”

$$f(p) = \phi \otimes \psi \cong \exp\left(\frac{p}{\tau}\right) \cdot \left[ 1 - \text{ferr} \left( \frac{p - p_0 + \frac{\sigma_{\text{pf}}^2}{\tau} - s \cdot \tau}{\sqrt{2} \sigma_{\text{pf}}} \right) \right]$$

where

$$\tau = \text{coef} \cdot \sqrt{A_{\text{PF}} \cdot E_S} / \beta,$$

$$\sigma_{\text{pf}}^2 = \beta \sigma_0^2 \frac{A_{\text{PF}} (A_P - A_{\text{PF}})}{A_P - 1}$$

$E_S$  is the energy spent to split the projectile  
(mass difference, surface energy excess)

$A_{\text{PF}}$  is the mass number of the prefragment.

Three parameters to fit:  $\sigma_0$ ,  $s$ , **coef**.

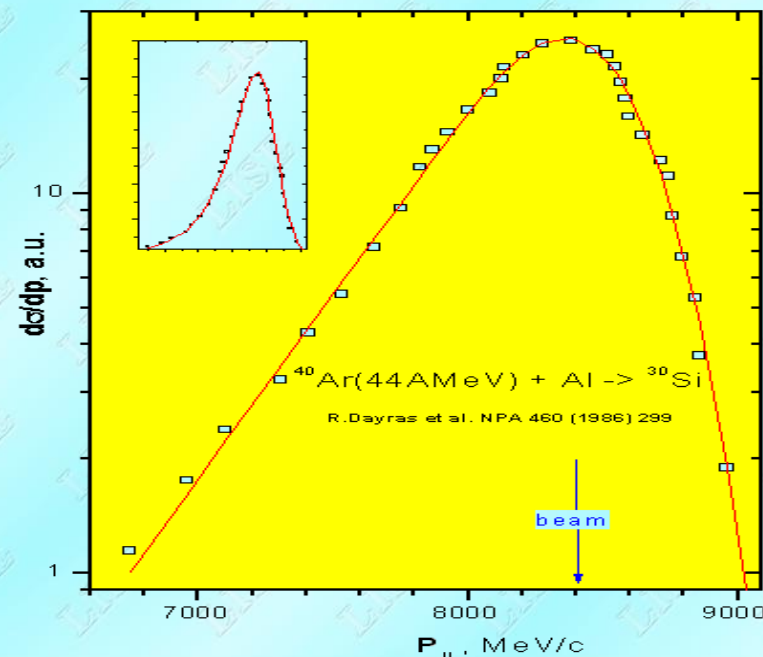
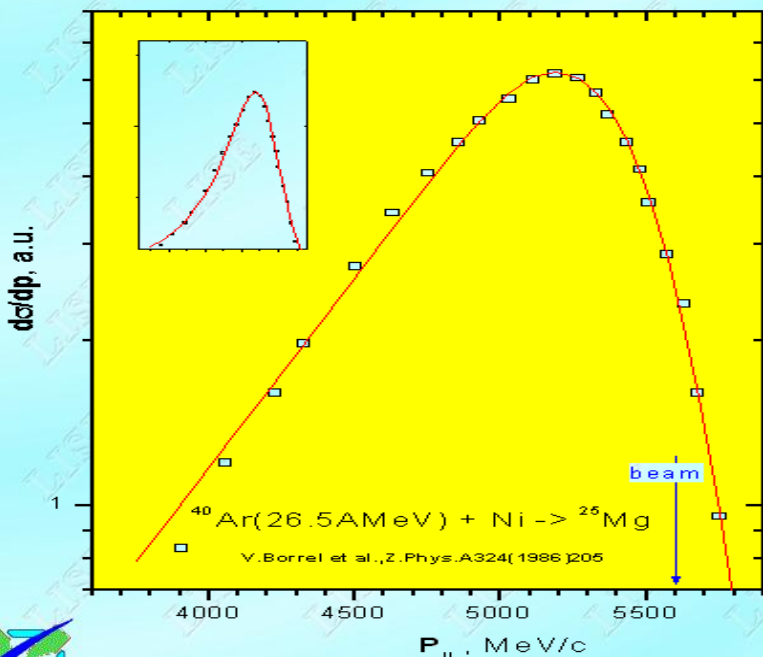
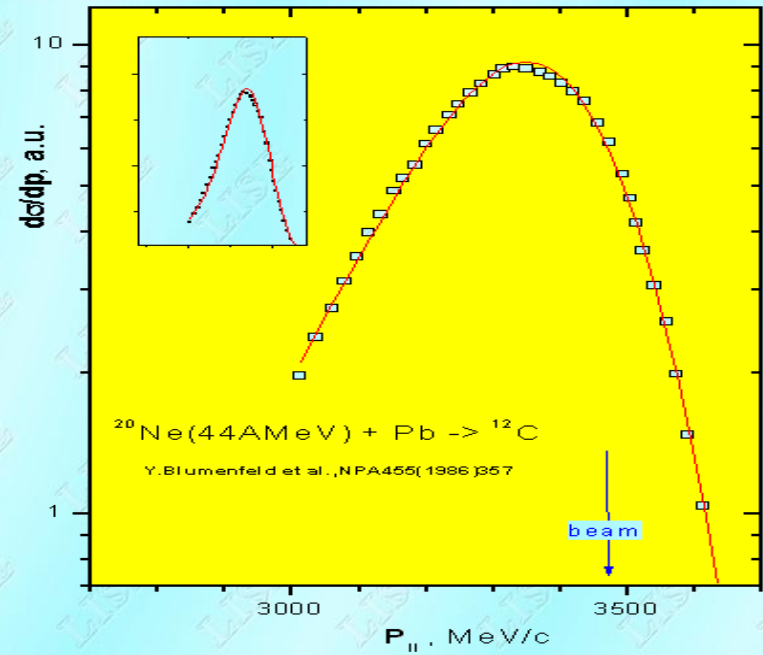
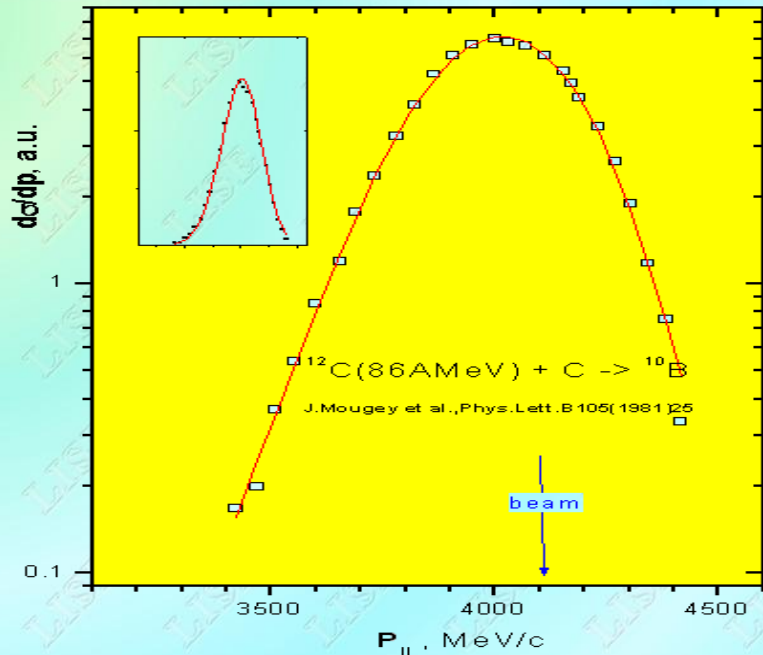
## Why is Universal?

- **Distribution Width**
- **Velocity ( $v_{\text{frag}}/v_{\text{beam}}$ )**
- **Low-energy tail**

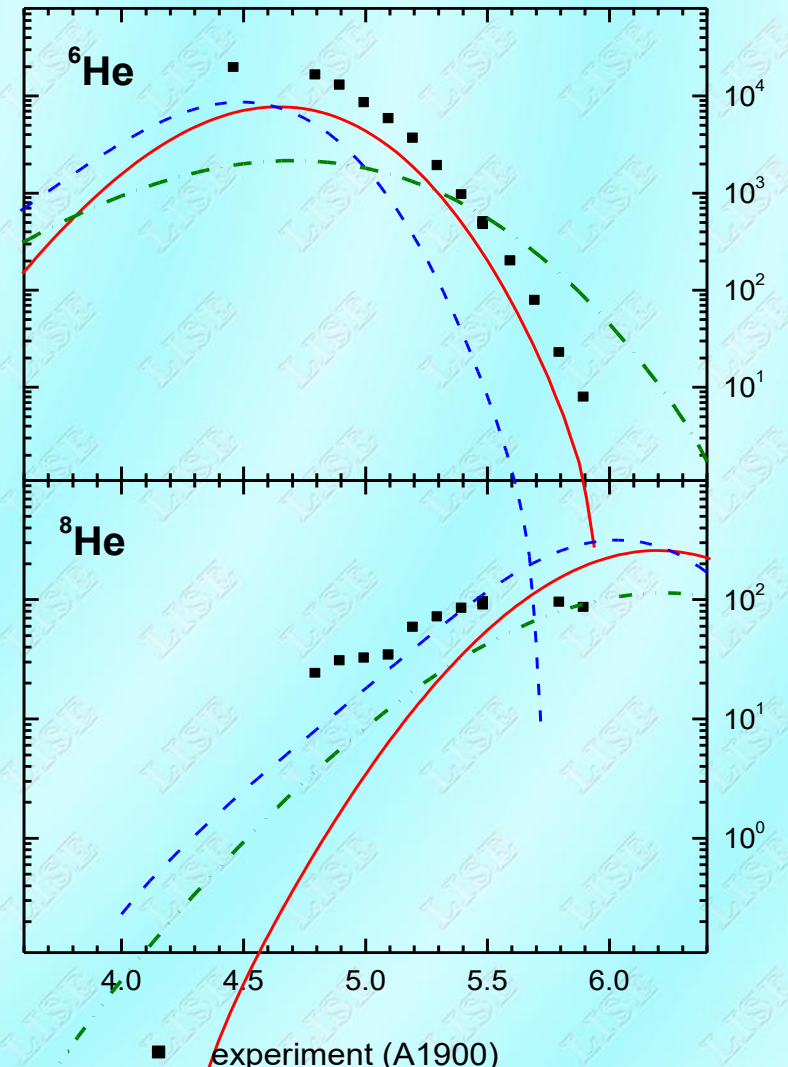
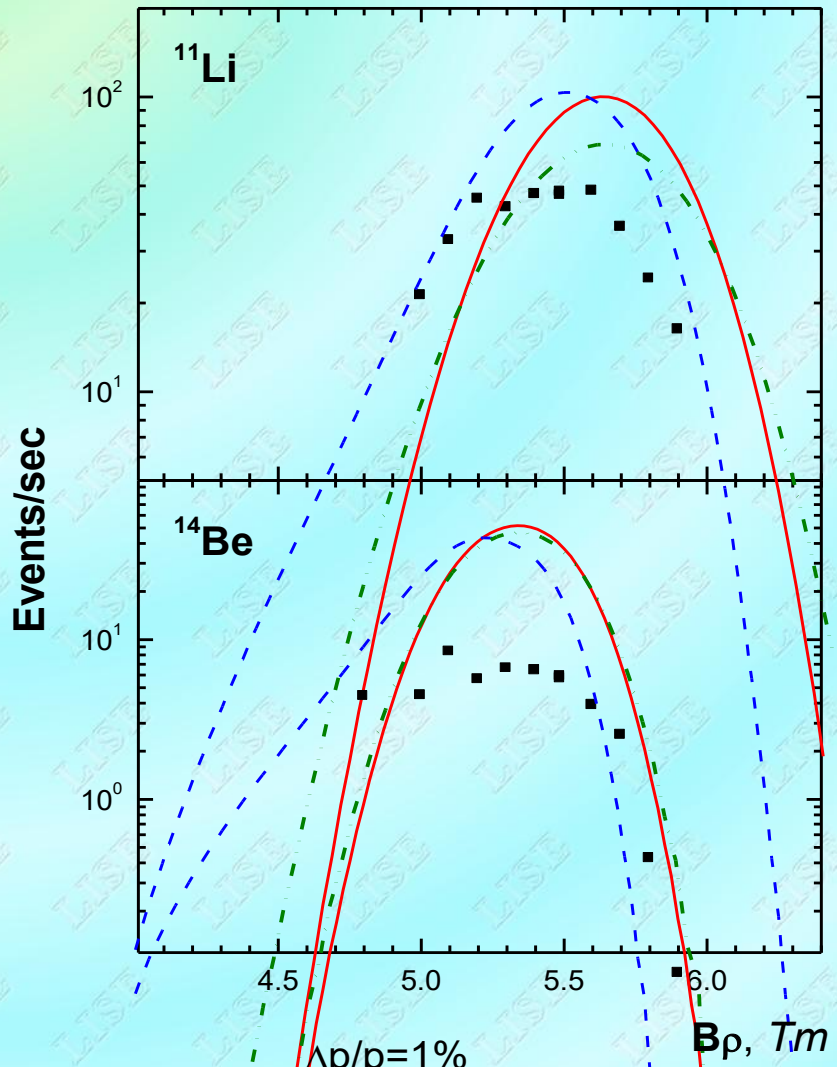
## Calculation steps

1. **Search the more probable prefragment for a given fragment.**
2. **Calculation of energy surface excess for the prefragment**  
[J.Gosset et al., Pys.Rev.C 16 (1977) 629]
3. **Calculation of Q-value using the database of mass.**





# A1900 / NSCL : $^{18}\text{O}$ (120MeV/u, 1pna) + $\text{Be}$ (1166 mg/cm<sup>2</sup>)

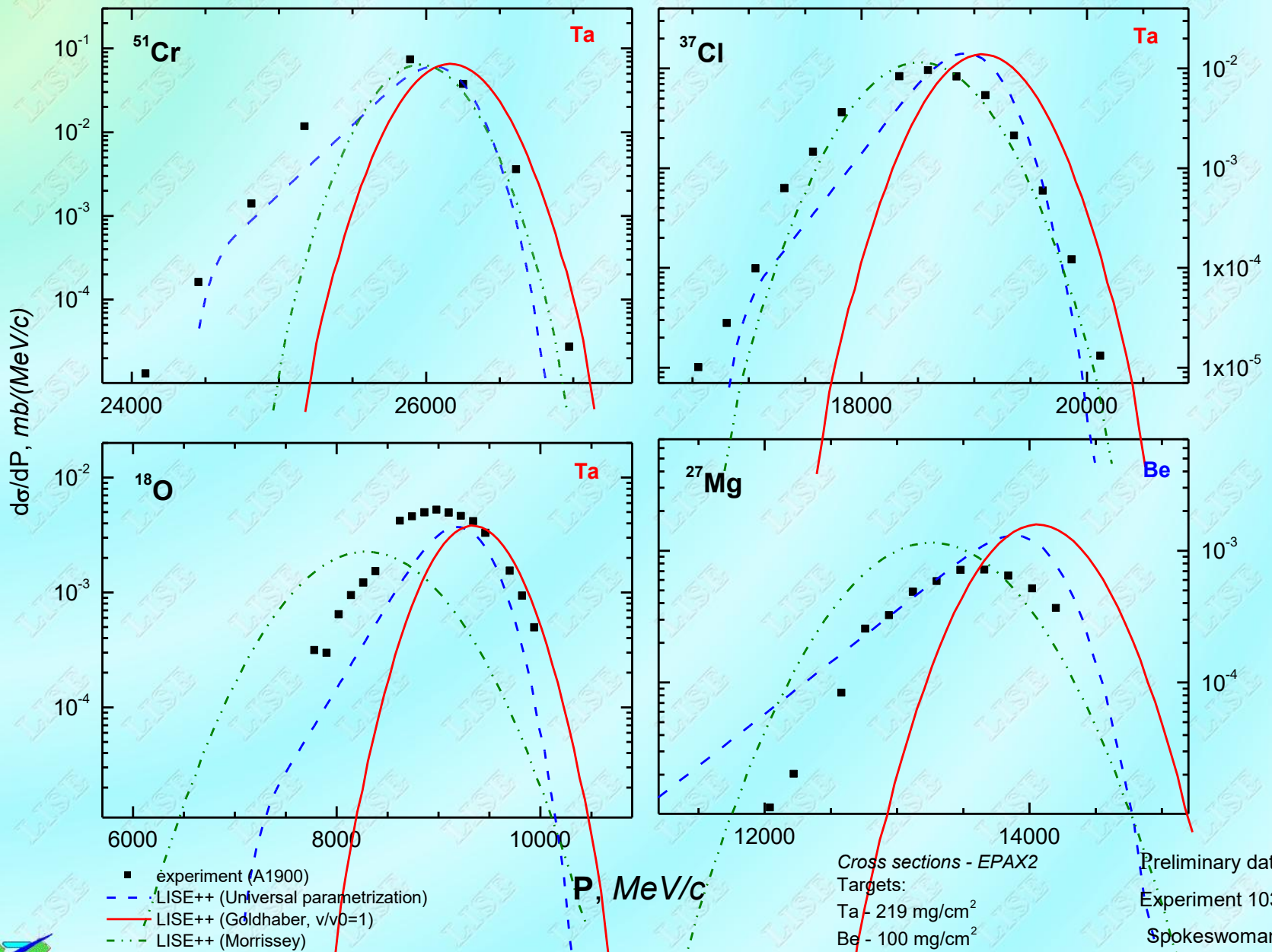


- experiment (A1900)
- LISE (Goldhaber formula)
- - - LISE (Universal parametrization)
- · - · - LISE (Morrissey parametrization)

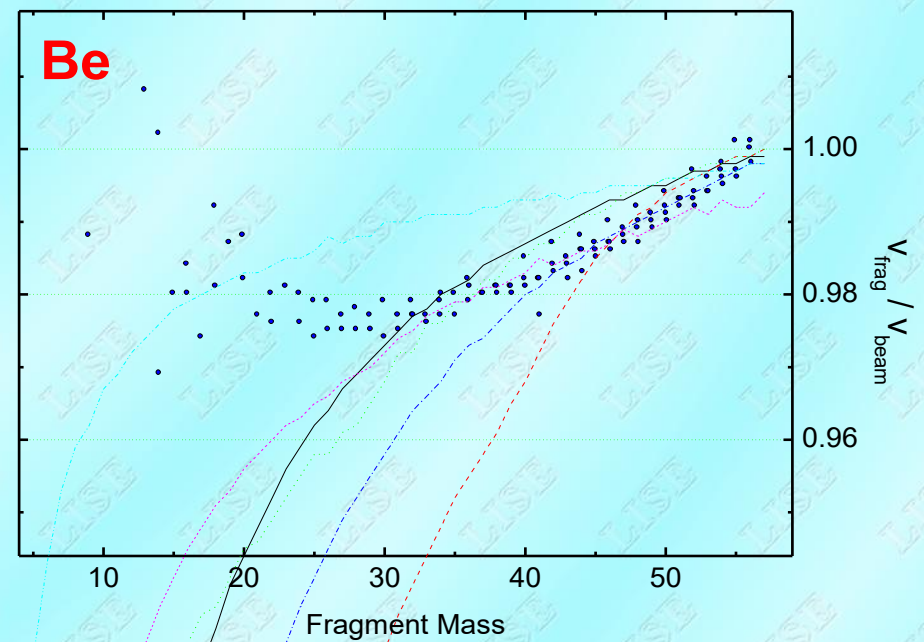
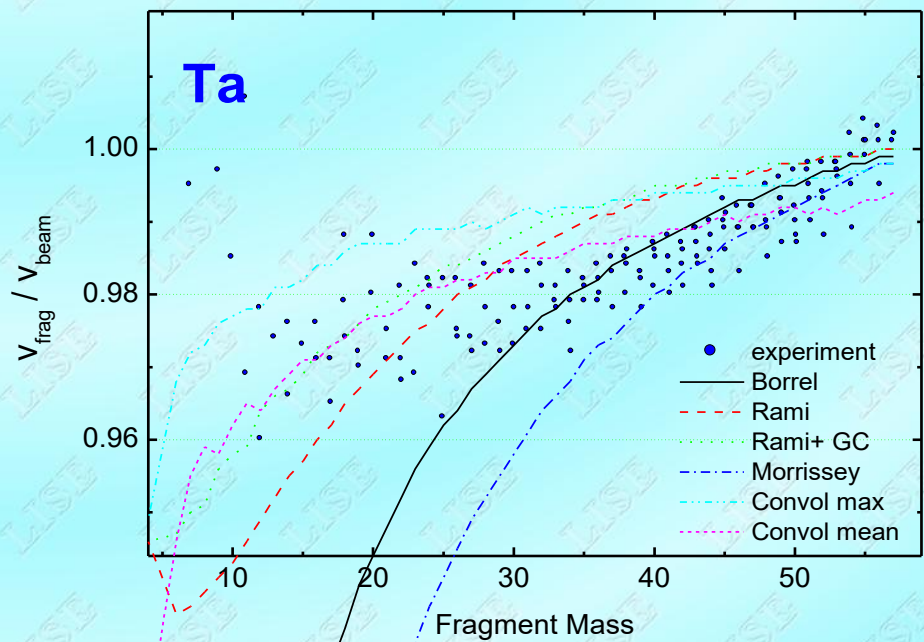
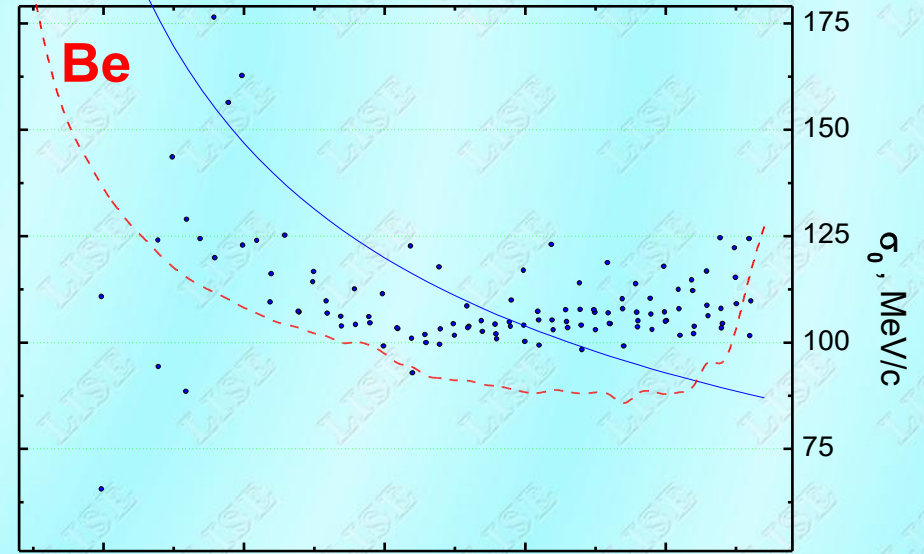
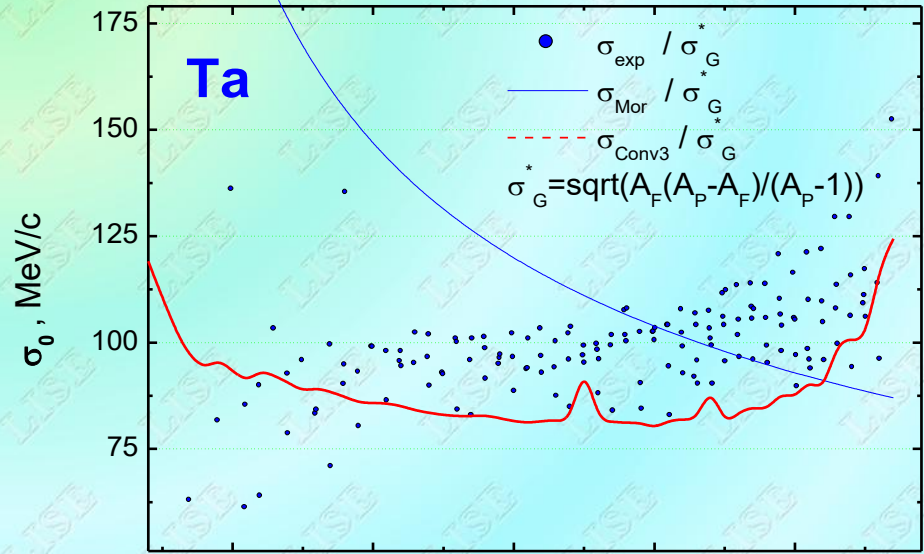
20% - is the systematical error of the beam current measurement  
 Cross sections - EPAX2



# A1900 / NSCL : $^{58}\text{Ni}$ (140 MeV/u) + Be, Ta



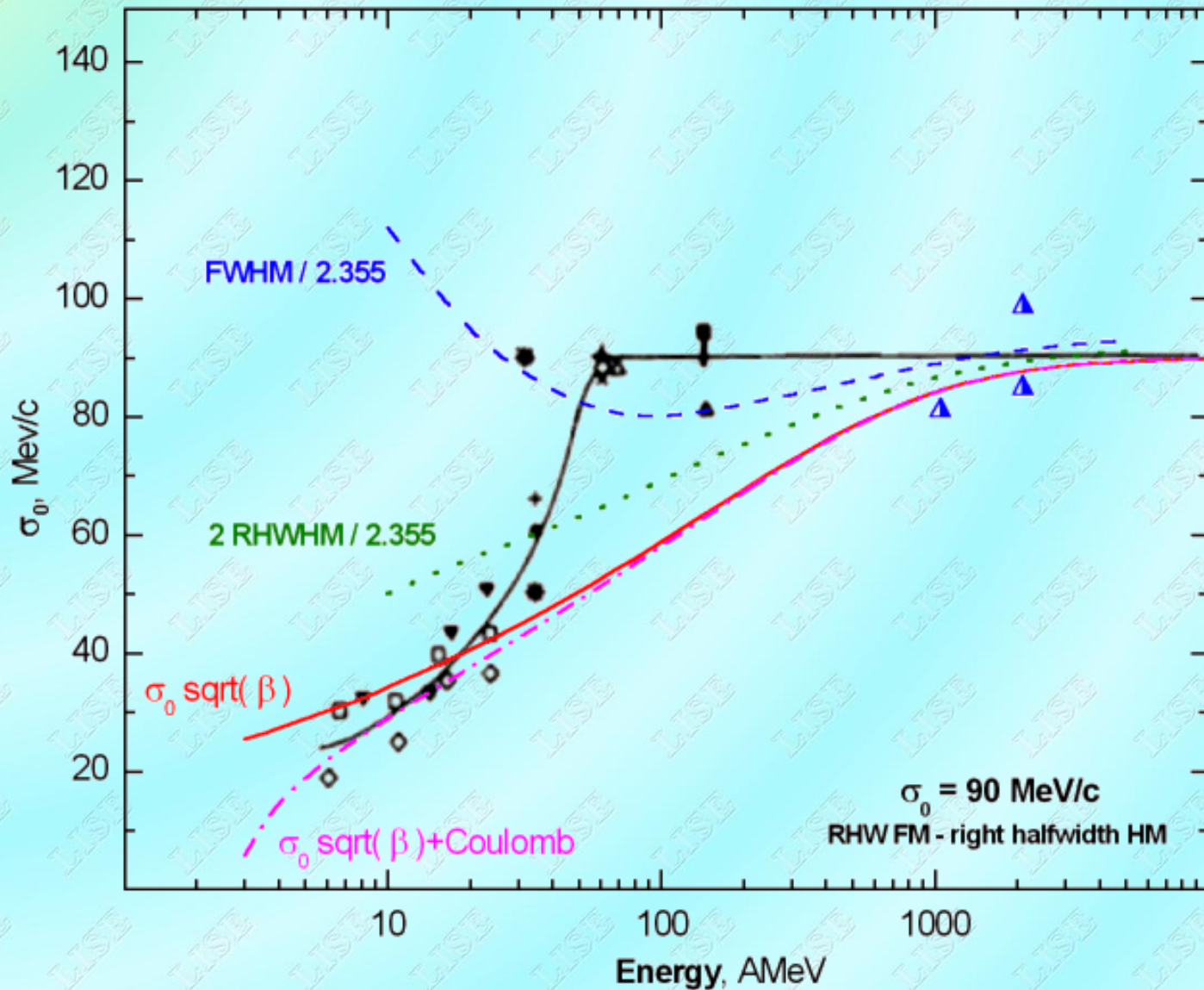
# A1900 / NSCL : $^{58}\text{Ni}$ (140 MeV/u) + Be,Ta



calculations have been done for more probable  $Z(A)$



# Momentum distribution width systematization



# Summary

- A model for fragment momentum distributions was developed as a function of a projectile energy.
- Analysis of several experimental studies has been performed to obtain coefficients of the Universal Parameterization, which allows to overcome drawbacks inherent to Goldhaber's and Morrissey's models.
- The Universal parameterization is incorporated in the LISE++ code for fragment transmission calculation.
- Comparisons with recent experimental data in the energy region of 120-140 MeV/u for various combinations of primary beams and targets are presented.

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