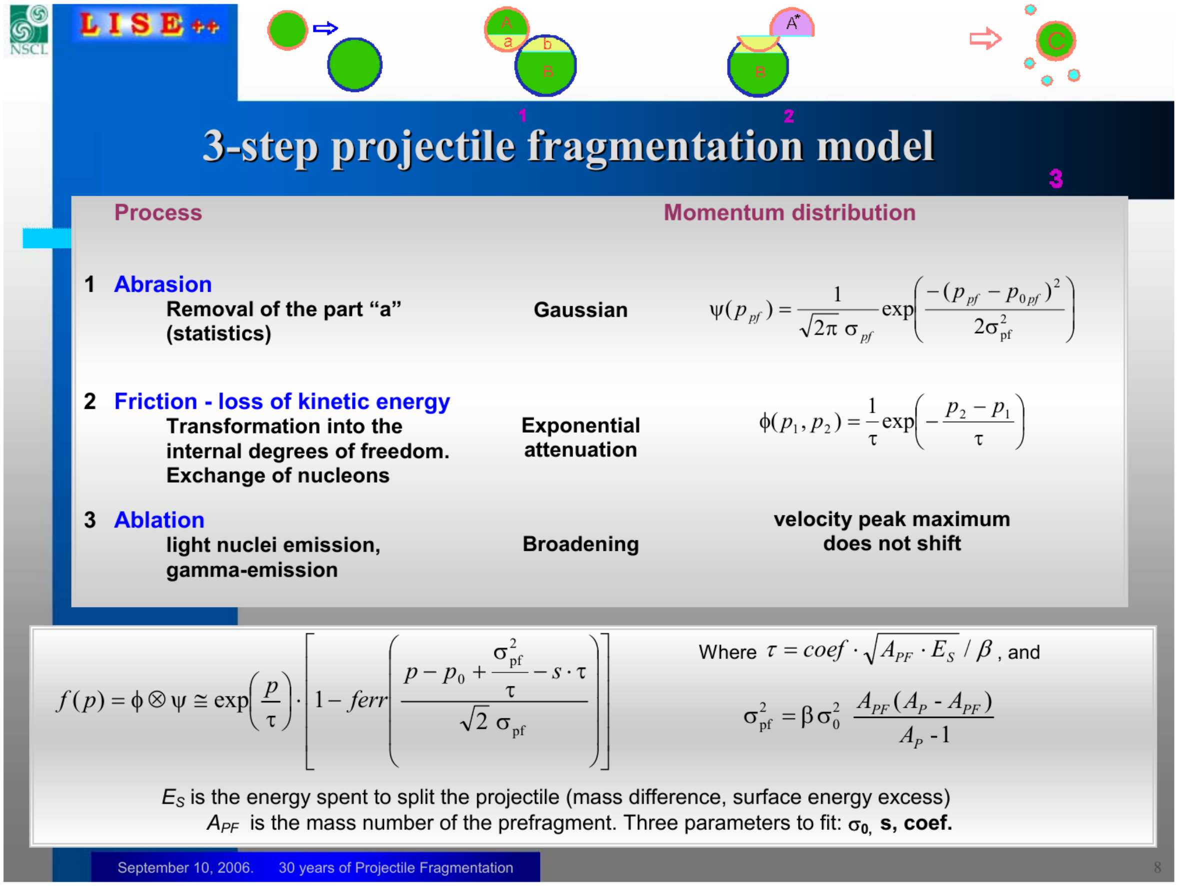
Universal parameterization
(Convolution method) in LISE^{++}

Detailed physics and implementation notes, including the search for the initial prefragment

Prepared from the 2004 Nuclear Physics A article, the original LISE v4.9 notes, the 2006 projectile-fragmentation presentation, the 2019 revision presentation, and later LISE^{++} version notes.



3-step projectile fragmentation model

Process		Momentum distribution
1 **Abrasion** Removal of the part “a” (statistics)	Gaussian	$\psi(p_{pf}) = \frac{1}{\sqrt{2\pi}\,\sigma_{pf}} \exp\left(\frac{-(p_{pf}-p_{0pf})^2}{2\sigma_{pf}^2}\right)$
2 **Friction - loss of kinetic energy** Transformation into the internal degrees of freedom. Exchange of nucleons	Exponential attenuation	$\phi(p_1, p_2) = \frac{1}{\tau} \exp\left(-\frac{p_2 - p_1}{\tau}\right)$
3 **Ablation** light nuclei emission, gamma-emission	Broadening	velocity peak maximum does not shift

$$f(p) = \phi \otimes \psi \cong \exp\left(\frac{p}{\tau}\right) \cdot \left[1 - ferr\left(\frac{p - p_0 + \frac{\sigma_{pf}^2}{\tau} - s \cdot \tau}{\sqrt{2}\,\sigma_{pf}}\right)\right]$$

Where $\tau = coef \cdot \sqrt{A_{PF} \cdot E_S} / \beta$, and

$$\sigma_{pf}^2 = \beta \sigma_0^2 \frac{A_{PF}(A_P - A_{PF})}{A_P - 1}$$

E_S is the energy spent to split the projectile (mass difference, surface energy excess)
A_{PF} is the mass number of the prefragment. Three parameters to fit: σ_0, **s, coef.**

September 10, 2006. 30 years of Projectile Fragmentation

8

Source figure. The 2006 presentation summarizes the three-step projectile-fragmentation picture used by the Universal parameterization: abrasion, friction, and ablation.

Contents

Document note

The historical LISE pages use both “parametrization” and “parameterization.” This document uses “parameterization,” while retaining the original name “Universal parameterization” and the LISE^{++} GUI terminology “Convolution method.”

1. Purpose and scope

The Universal parameterization, also called the Convolution method in LISE^{++}, is a semi-empirical model for the longitudinal momentum distribution of projectile-fragmentation residues. In LISE^{++} it is used primarily for transmission calculations: once a production cross section has been assigned by EPAX, abrasion-ablation, a user table, or another production mechanism, the momentum distribution determines how much of that yield is accepted by the separator optics.

The model was introduced because a simple Goldhaber Gaussian distribution is not flexible enough at low and intermediate projectile energies. The Universal parameterization keeps the statistical Goldhaber-type core but folds it with an exponential attenuation term that represents dissipative participant-spectator friction. The result can reproduce three observables with one connected model: the distribution width, the fragment-to-projectile velocity ratio, and the low-momentum tail.

This document emphasizes one point that is easy to miss in a short description: before the Convolution distribution can be calculated for a final residue, LISE^{++} must identify the most probable initial prefragment. That prefragment fixes the mass entering the Gaussian width and the separation/excitation-energy scale entering the exponential friction term.

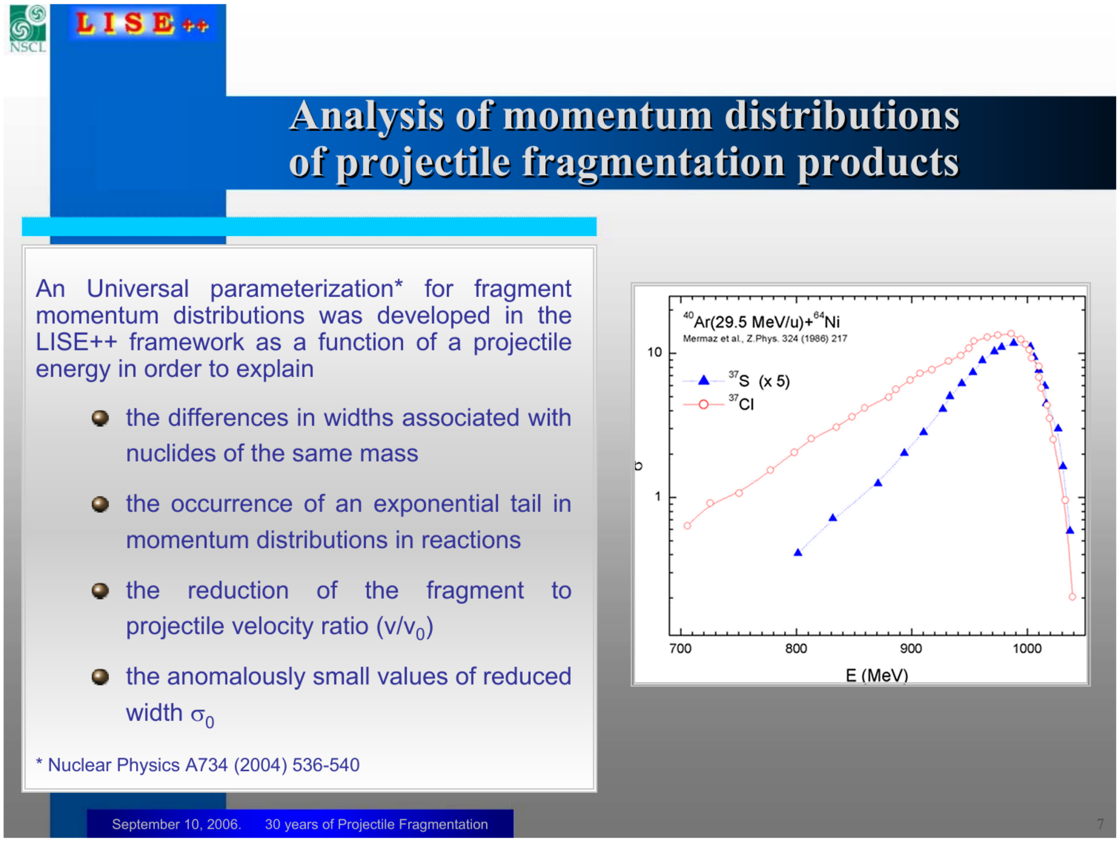
2. Why the Universal parameterization was introduced

At relativistic energies, many projectile-fragment momentum distributions are close to Gaussian and centered near the projectile velocity. The Goldhaber model describes this by assuming random removal of nucleons from a projectile with zero total internal momentum. In this limit, the width follows a parabolic dependence on projectile and fragment masses, with a reduced width related to the Fermi momentum.

However, the experimental situation at low and intermediate energies is more complicated. The original LISE v4.9 notes and the 2004 article identify several deficiencies of a pure Goldhaber treatment:

- Fragments with the same mass number may have different observed longitudinal widths
- The reduced width σ_0 becomes anomalously small at lower beam energies
- The mean fragment velocity can be lower than the projectile velocity, especially at lower energies
- The momentum spectrum can develop a pronounced low-momentum exponential tail
- The apparent extracted width depends on whether the full spectrum or only the Gaussian-like high-momentum side is used

Earlier LISE versions therefore contained several separate ingredients: empirical width systematics such as Morrissey, Friedman separation-energy-based widths with Coulomb corrections, and velocity-shift prescriptions based on removed binding energy or surface-energy excess. The Universal parameterization was created to replace this patchwork by a single distribution model in which the width, velocity shift, and exponential tail are generated together.



Analysis of momentum distributions of projectile fragmentation products

An Universal parameterization* for fragment momentum distributions was developed in the LISE++ framework as a function of a projectile energy in order to explain

- the differences in widths associated with nuclides of the same mass
- the occurrence of an exponential tail in momentum distributions in reactions
- the reduction of the fragment to projectile velocity ratio (v/v_0)
- the anomalously small values of reduced width σ_0

* Nuclear Physics A734 (2004) 536-540

September 10, 2006. 30 years of Projectile Fragmentation

Figure 1. Motivation for the Universal parameterization from the 2006 presentation. The model was introduced to describe isotope-dependent widths, low-energy exponential tails, reduced fragment velocity, and anomalously small reduced widths.

3. Physics picture: abrasion, friction, and ablation

The Universal parameterization follows the three-step projectile-fragmentation picture. It is not a full microscopic reaction simulation; rather, it encodes the main kinematic consequences of the fragmentation process in a form suitable for fast separator-acceptance calculations.

3.1 Abrasion: creation of an excited prefragment

In the first stage, the projectile overlaps with the target. Nucleons in the participant zone are removed, while the spectator part of the projectile survives as an excited prefragment. The prefragment momentum distribution is assumed to have a Gaussian form, following the same statistical logic as the Goldhaber model. The width is controlled by the number of removed nucleons and by the prefragment mass.

The important LISE^{++} distinction is that the observed final fragment is usually not this initial prefragment. The final residue appears after evaporation of neutrons, protons, light clusters, and gamma emission. Therefore the prefragment used for the Convolution calculation must be searched for.

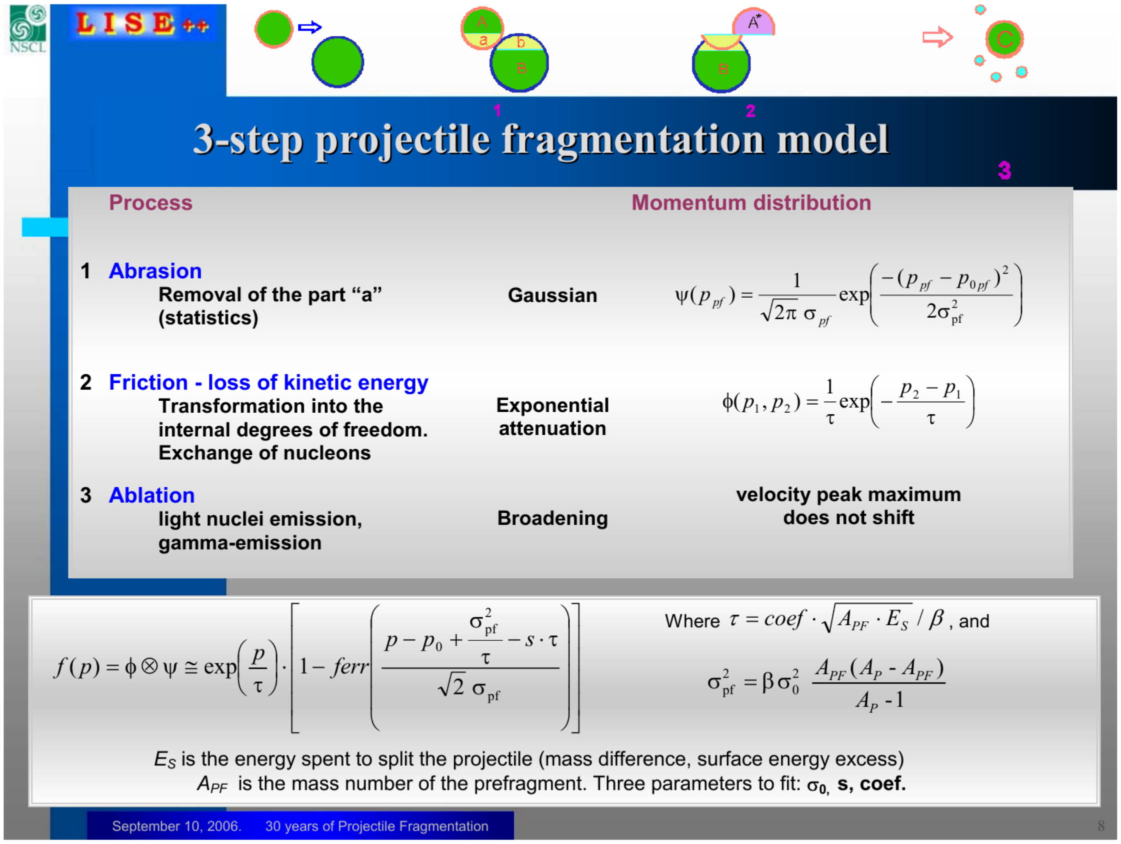
3.2 Friction: kinetic-energy loss and exponential attenuation

The second stage is the key addition of the Universal parameterization. The projectile spectator does not simply continue at the projectile velocity. It interacts with participant matter, loses kinetic energy, exchanges nucleons, and converts part of the ordered longitudinal motion into internal degrees of freedom. In the model this dissipative process is represented by an exponential attenuation in momentum.

This exponential term produces the asymmetric low-momentum tail. It also shifts the effective distribution toward lower fragment velocities. As projectile energy increases, the relative importance of the friction term decreases and the distribution approaches the high-energy Gaussian limit.

3.3 Ablation: evaporation and broadening

In the third stage, the excited prefragment decays. Evaporation broadens the final velocity distribution and changes the mass and charge of the residue, but in the simplified Convolution description the ablation stage is treated mainly as broadening, not as a separate shift of the velocity-peak maximum. This makes the method computationally efficient while preserving the features most relevant for magnetic-rigidity acceptance.



3-step projectile fragmentation model

Process		Momentum distribution
1 **Abrasion** Removal of the part “a” (statistics)	Gaussian	$\psi(p_{pf}) = \frac{1}{\sqrt{2\pi}\,\sigma_{pf}} \exp\left(\frac{-(p_{pf} - p_{0pf})^2}{2\sigma_{pf}^2} \right)$
2 **Friction - loss of kinetic energy** Transformation into the internal degrees of freedom. Exchange of nucleons	Exponential attenuation	$\phi(p_1, p_2) = \frac{1}{\tau} \exp\left(-\frac{p_2 - p_1}{\tau} \right)$
3 **Ablation** light nuclei emission, gamma-emission	Broadening	velocity peak maximum does not shift

$$f(p) = \phi \otimes \psi \cong \exp\left(\frac{p}{\tau}\right) \cdot \left[1 - ferr\left(\frac{p - p_0 + \frac{\sigma_{pf}^2}{\tau} - s \cdot \tau}{\sqrt{2}\,\sigma_{pf}} \right) \right]$$

Where $\tau = coef \cdot \sqrt{A_{PF} \cdot E_S} / \beta$, and

$$\sigma_{pf}^2 = \beta \sigma_0^2 \, \frac{A_{PF}(A_P - A_{PF})}{A_P - 1}$$

E_S is the energy spent to split the projectile (mass difference, surface energy excess)

A_{PF} is the mass number of the prefragment. Three parameters to fit: σ_0, **s, coef.**

September 10, 2006. 30 years of Projectile Fragmentation

Figure 2. Three-step model from the 2006 presentation: Gaussian abrasion distribution, exponential friction attenuation, and ablation broadening.

4. Mathematical form of the Convolution method

The final distribution is obtained by folding a Gaussian prefragment distribution with an exponential attenuation function. In compact notation:

$$f(p) = \varphi \otimes \psi$$

where ψ is the Gaussian prefragment momentum distribution and ϕ is the exponential attenuation caused by friction.

4.1 Gaussian prefragment distribution

The prefragment distribution is written as a Gaussian centered at the momentum corresponding to projectile velocity:

$$\psi(p\_PF) = 1 / (\sqrt{(2\pi)}\ \sigma\_PF) \cdot \exp[-(p\_PF - p_0PF)^2 / (2\ \sigma\_PF^2)]$$

The prefragment width is based on the Goldhaber mass dependence but uses the mass of the prefragment, not necessarily the final fragment:

$$\sigma\_PF^2 = \beta \cdot \sigma_0^2 \cdot A\_PF(A\_P - A\_PF) / (A\_P - 1)$$

Here A_P is the projectile mass, A_PF is the prefragment mass, β is the projectile velocity in units of c, and σ_0 is the fitted reduced-width parameter. Historical LISE documentation and the 2004 publication use σ_conv = 91.5 MeV/c as the fitted width scale for the published parameter set.

4.2 Exponential attenuation

The exponential attenuation is written as

$$\varphi(p_1, p_2) = (1 / \tau) \cdot \exp[-(p_2 - p_1) / \tau]$$

where τ is the friction scale. In the LISE Universal parameterization, τ is proportional to the energy needed to split or excite the projectile into the selected prefragment:

$$\tau = coef \cdot \sqrt{A\_PF} \cdot E\_S / \beta$$

The friction scale therefore grows with the separation/excitation-energy scale E_S and with √A_PF, but decreases with projectile velocity. This is the physical reason the exponential tail is strongest at lower projectile energies and becomes less important at relativistic energies.

4.3 Folded distribution

After convolution, LISE uses an analytic expression containing an exponential factor and an error-function term. The 2006 presentation writes it in the approximate form:

$$f(p) \approx \exp(p / \tau) \cdot [1 - ferr((p - p_0 + \sigma\_PF^2/\tau - s\tau) / (\sqrt{2}\ \sigma\_PF))]$$

The parameter s appears as an empirical correction inside the error-function term. It compensates the observed velocity shift that is larger than expected from the simplest energy-loss argument. Together, σ_0, s, and coef define the shape and position of the calculated distribution.

Interpretation

The Gaussian part keeps the statistical abrasion core. The exponential part adds dissipative physics. The error-function form appears because the convolution of a Gaussian with an exponential has an exponentially modified Gaussian form. This is exactly the mathematical structure needed to generate a Gaussian-like high-momentum side and a long low-momentum tail.

5. Separation energy and fitted coefficients

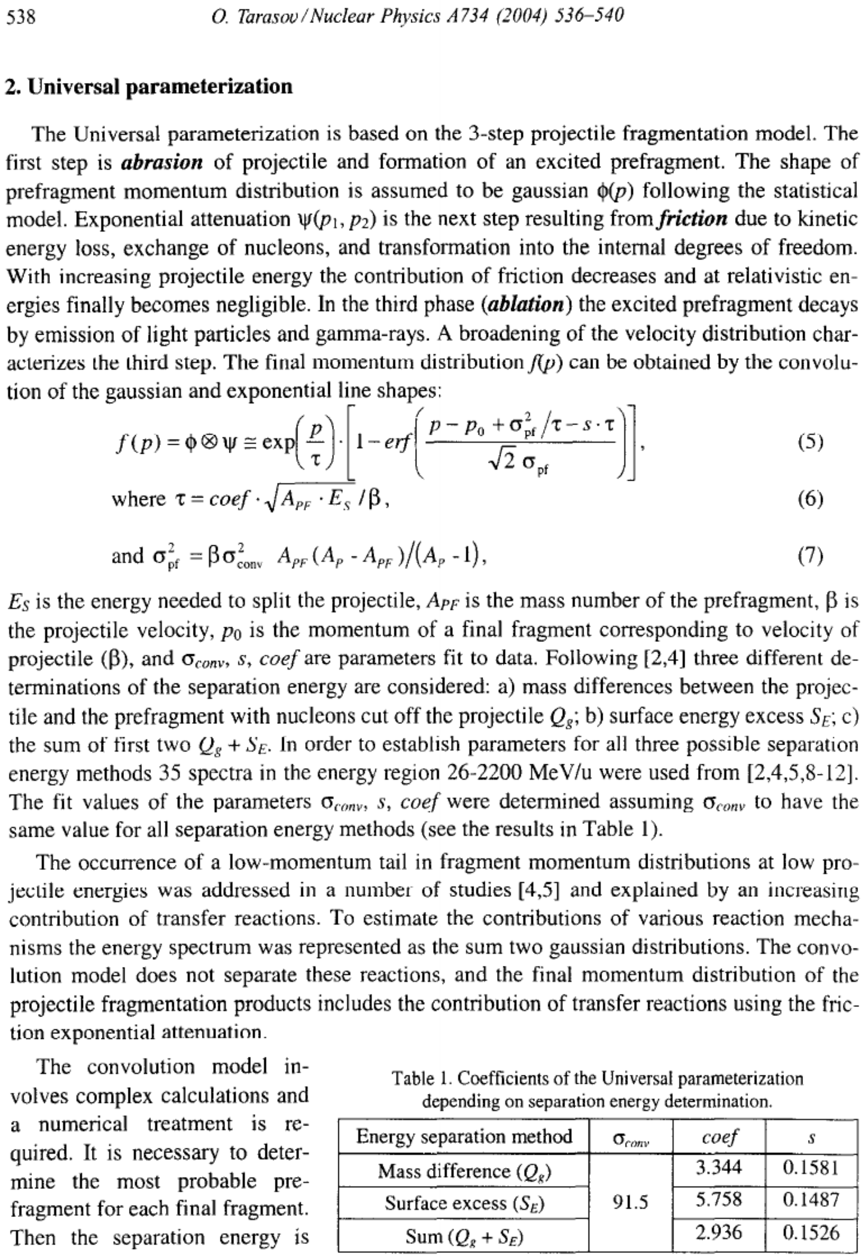
The energy E_S is one of the central inputs. It represents the energy spent to split the projectile into the prefragment and the removed participant part, including the excitation or surface cost associated with that split. Historically, three prescriptions were used:

- Mass difference Q_g between the projectile and the prefragment with the removed nucleons
- Surface-energy excess SE, based on the contact surface between the abraded zone and the remaining fragment
- The sum Q_g + SE

The 2004 Nuclear Physics A article fitted 35 spectra in the energy range 26-2200 MeV/u. The same σ_conv value was used for all three E_S prescriptions, while coef and s depended on the separation-energy definition.

E_S prescription	σ_conv (MeV/c)	coef	s	Physical meaning
Mass difference Q_g	91.5	3.344	0.1581	Cluster/mass-difference energy for removing the participant part
Surface excess SE	91.5	5.758	0.1487	Surface-energy cost of the contact/cut geometry
Q_g + SE	91.5	2.936	0.1526	Combined mass-difference and surface-energy cost

The original LISE v4.9 notes also describe the model as requiring two stages: formation of the prefragment distribution with kinetic-energy loss, followed by evaporation of the prefragment. The same notes state that the prefragment mass gave the best approximation in the τ expression, and that τ had to be inversely proportional to fragment velocity to reproduce the amplitude of the exponential tail.



538 O. Tarasov / Nuclear Physics A734 (2004) 536–540

2. Universal parameterization

The Universal parameterization is based on the 3-step projectile fragmentation model. The first step is ***abrasion*** of projectile and formation of an excited prefragment. The shape of prefragment momentum distribution is assumed to be gaussian $\phi(p)$ following the statistical model. Exponential attenuation $\psi(p_1, p_2)$ is the next step resulting from ***friction*** due to kinetic energy loss, exchange of nucleons, and transformation into the internal degrees of freedom. With increasing projectile energy the contribution of friction decreases and at relativistic energies finally becomes negligible. In the third phase (***ablation***) the excited prefragment decays by emission of light particles and gamma-rays. A broadening of the velocity distribution characterizes the third step. The final momentum distribution $f(p)$ can be obtained by the convolution of the gaussian and exponential line shapes:

$$f(p) = \phi \otimes \psi \cong \exp\left(\frac{p}{\tau}\right) \cdot \left[1 - erf\left(\frac{p - p_0 + \sigma_{\text{pf}}^2/\tau - s \cdot \tau}{\sqrt{2}\,\sigma_{\text{pf}}}\right)\right], \qquad (5)$$

where $\tau = coef \cdot \sqrt{A_{PF} \cdot E_S} / \beta, \qquad (6)$

and $\sigma_{\text{pf}}^2 = \beta \sigma_{\text{conv}}^2 \; A_{PF}(A_P - A_{PF}) / (A_P - 1), \qquad (7)$

E_S is the energy needed to split the projectile, A_{PF} is the mass number of the prefragment, β is the projectile velocity, p_0 is the momentum of a final fragment corresponding to velocity of projectile (β), and σ_{conv}, s, $coef$ are parameters fit to data. Following [2,4] three different determinations of the separation energy are considered: a) mass differences between the projectile and the prefragment with nucleons cut off the projectile Q_g; b) surface energy excess S_E; c) the sum of first two $Q_g + S_E$. In order to establish parameters for all three possible separation energy methods 35 spectra in the energy region 26-2200 MeV/u were used from [2,4,5,8-12]. The fit values of the parameters σ_{conv}, s, $coef$ were determined assuming σ_{conv} to have the same value for all separation energy methods (see the results in Table 1).

The occurrence of a low-momentum tail in fragment momentum distributions at low projectile energies was addressed in a number of studies [4,5] and explained by an increasing contribution of transfer reactions. To estimate the contributions of various reaction mechanisms the energy spectrum was represented as the sum two gaussian distributions. The convolution model does not separate these reactions, and the final momentum distribution of the projectile fragmentation products includes the contribution of transfer reactions using the friction exponential attenuation.

Table 1. Coefficients of the Universal parameterization depending on separation energy determination.

Energy separation method	σ_{conv}	$coef$	s
Mass difference (Q_g)	91.5	3.344	0.1581
Surface excess (S_E)		5.758	0.1487
Sum ($Q_g + S_E$)		2.936	0.1526

The convolution model involves complex calculations and a numerical treatment is required. It is necessary to determine the most probable prefragment for each final fragment. Then the separation energy is

Figure 3. Page from the 2004 Nuclear Physics A article showing the Universal parameterization equations and the table of fitted coefficients.

6. Search for the initial prefragment

The search for the initial prefragment is a required internal step. The measured or requested isotope is the final residue after ablation, not necessarily the projectile spectator immediately after abrasion. Many parent prefragments can feed the same final nucleus through different evaporation paths. The Convolution method must choose the prefragment that is most probable and most relevant for the longitudinal momentum distribution.

6.1 Why the prefragment search affects the momentum distribution

The selected prefragment directly determines four key quantities:

- A_PF, which enters the Goldhaber-type width σ_PF
- E_S, which controls the friction scale τ
- The number of abraded nucleons ΔA = A_P - A_PF, which controls the excitation-energy scale
- The likely evaporation path from prefragment to final fragment

Using the final fragment mass instead of the prefragment mass would generally underestimate or misplace the initial distribution, because the final fragment has already lost particles in the ablation stage. This is especially important for very neutron-rich or proton-rich residues where only a restricted set of evaporation chains can feed the residue.

6.2 Historical search options

The 2019 revision notes explicitly separated Universal parameterization into two blocks: the prefragment search and the momentum-distribution Convolution model. Earlier search logic was found to be outdated because more probable prefragments calculated with the LISE^{++} abrasion-ablation model could differ drastically from the old Universal-parameterization search results.

The prefragment-search dialog shown in the 2019 presentation contains three search options:

Option	Meaning in the dialog	Comment
A	Search in the N/Z beam direction	Older geometrical/isospin-direction search; useful for historical reproduction but less connected to AA probabilities
B	Search a parent using emission widths W and EPAX cross sections	Uses evaporation/emission information with EPAX-like production weighting
C	Search a parent using emission widths W and abrasion initial cross section	Recommended direction in the revision because it ties the prefragment probability to abrasion physics

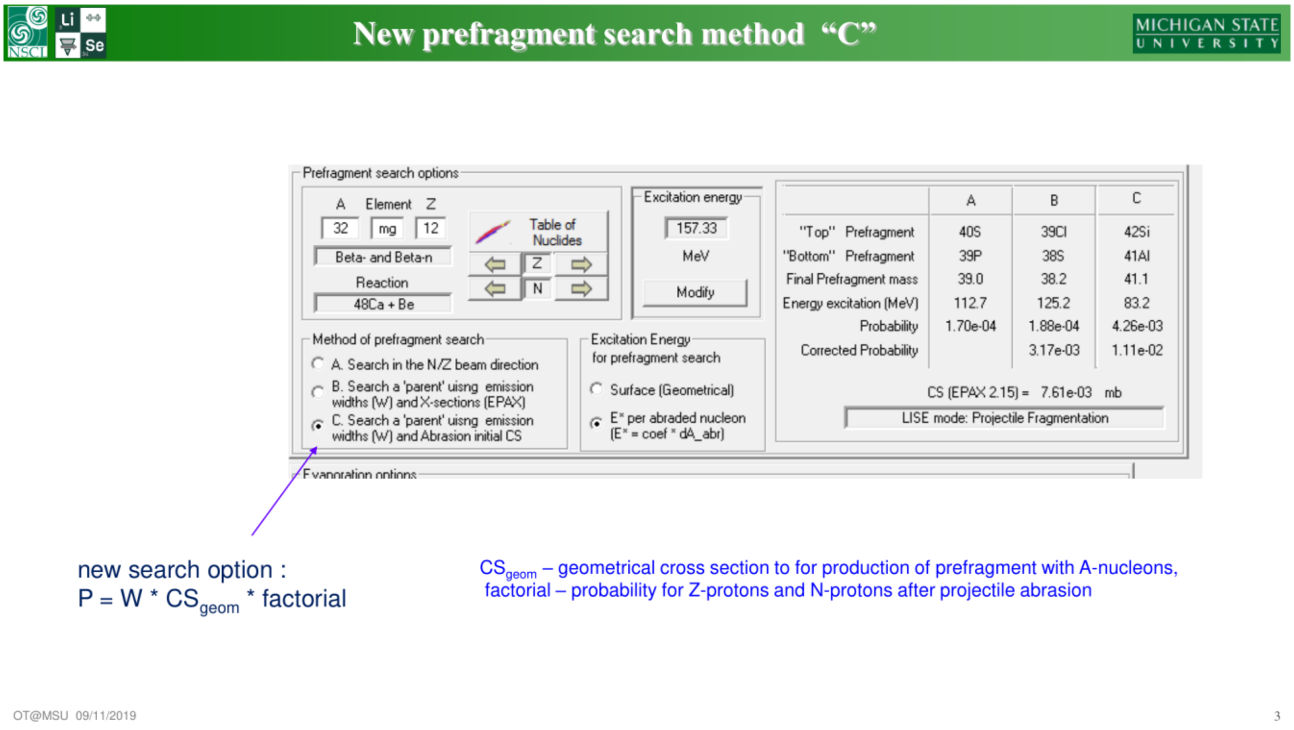
6.3 New search option C

The new option C uses the probability form

```
P = W · CS_geom · factorial
```

where W represents the evaporation/emission width weight for feeding the final fragment, CS_geom is the geometrical cross section for producing a prefragment with the selected number of abraded nucleons, and the factorial term is the combinatorial probability of obtaining the required proton and neutron content after projectile abrasion.

This is a physically important change. It means the prefragment search is no longer only a geometrical direction in the N/Z plane. It is weighted by an abrasion probability and by the probability that the candidate prefragment can evaporate into the requested final fragment.



New prefragment search method "C"

Prefragment search options

A: 32 · Element: mg · Z: 12

Beta- and Beta-n

Reaction: 48Ca + Be

Excitation energy: 157.33 MeV

	A	B	C
"Top" Prefragment	40S	39Cl	42Si
"Bottom" Prefragment	39P	38S	41Al
Final Prefragment mass	39.0	38.2	41.1
Energy excitation (MeV)	112.7	125.2	83.2
Probability	1.70e-04	1.88e-04	4.26e-03
Corrected Probability		3.17e-03	1.11e-02

CS (EPAX 2.15) = 7.61e-03 mb

LISE mode: Projectile Fragmentation

Method of prefragment search

- ○ A. Search in the N/Z beam direction
- ○ B. Search a 'parent' uisng emission widths (W) and X-sections (EPAX)
- ● C. Search a 'parent' uisng emission widths (W) and Abrasion initial CS

Excitation Energy for prefragment search

- ○ Surface (Geometrical)
- ● E* per abraded nucleon (E* = coef * dA_abr)

new search option :
$P = W * CS_{geom} * factorial$

CS_{geom} – geometrical cross section to for production of prefragment with A-nucleons,
factorial – probability for Z-protons and N-protons after projectile abrasion

OT@MSU 09/11/2019

3

Figure 4. 2019 revision slide introducing prefragment-search option C with P = W · CS_geom · factorial.

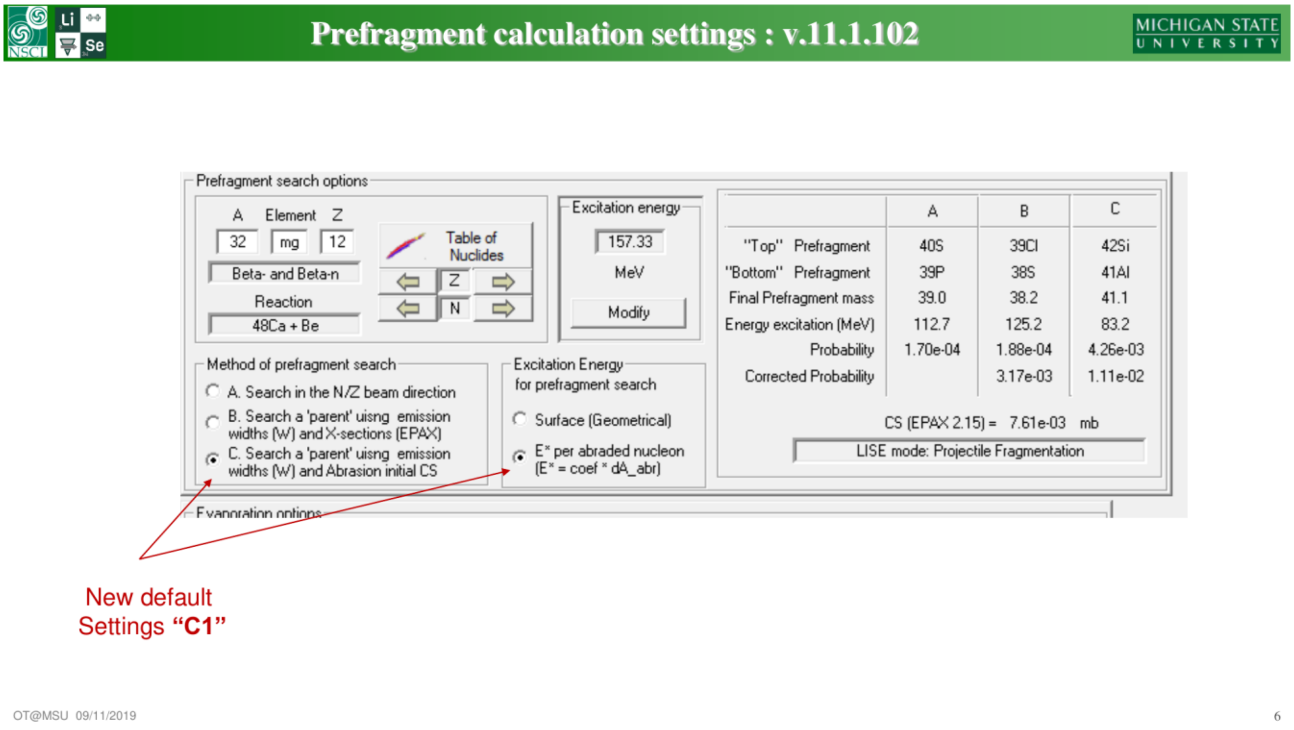
6.4 Excitation-energy model for the prefragment search

The earlier prefragment search used the geometrical dSurface excitation-energy model. The 2019 revision noted that this was inconsistent with the excitation-energy treatment used in the LISE^{++} abrasion-ablation cross-section calculations, where the Gaimard-Schmidt-type picture is usually central. A new search option was therefore added: E* per abraded nucleon.

In the updated dialog, this appears as an excitation-energy option for the prefragment search:

- Surface (Geometrical), corresponding to the older dSurface model
- E* per abraded nucleon, written in the dialog as E* = coef · ΔA_abr

The combined default recommended in v11.1.102 is C1: search option C plus the E* per abraded nucleon excitation-energy treatment.



Prefragment calculation settings : v.11.1.102

Prefragment search options

A 32 Element mg Z 12

Beta- and Beta-n

Reaction 48Ca + Be

Excitation energy 157.33 MeV

Method of prefragment search
- ○ A. Search in the N/Z beam direction
- ○ B. Search a 'parent' uisng emission widths (W) and X-sections (EPAX)
- ● C. Search a 'parent' uisng emission widths (W) and Abrasion initial CS

Excitation Energy for prefragment search
- ○ Surface (Geometrical)
- ● E* per abraded nucleon (E* = coef * dA_abr)

	A	B	C
"Top" Prefragment	40S	39Cl	42Si
"Bottom" Prefragment	39P	38S	41Al
Final Prefragment mass	39.0	38.2	41.1
Energy excitation (MeV)	112.7	125.2	83.2
Probability	1.70e-04	1.88e-04	4.26e-03
Corrected Probability		3.17e-03	1.11e-02

CS (EPAX 2.15) = 7.61e-03 mb

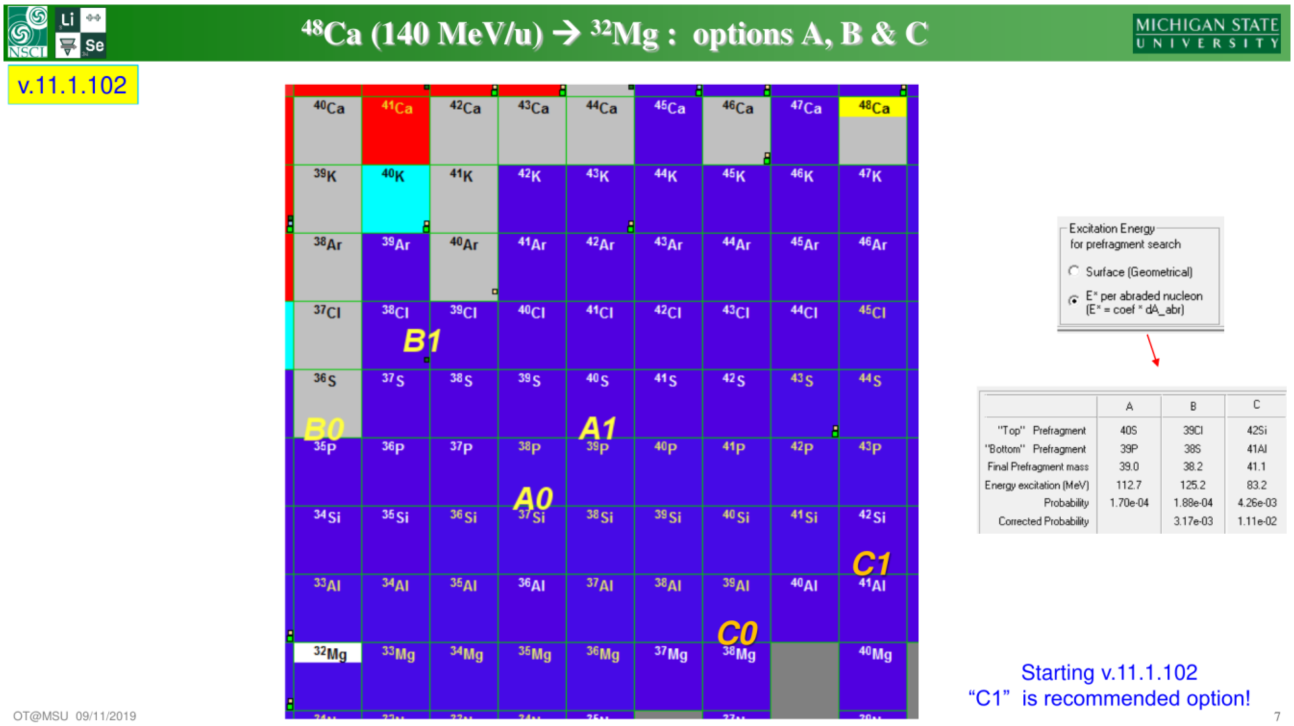
LISE mode: Projectile Fragmentation

New default Settings **"C1"**

OT@MSU 09/11/2019

6

Figure 5. 2019 revision slide showing the new default prefragment calculation settings, C1, in v11.1.102.



^{48}Ca (140 MeV/u) → ^{32}Mg : options A, B & C

v.11.1.102

Excitation Energy for prefragment search
- ○ Surface (Geometrical)
- ● E* per abraded nucleon (E* = coef * dA_abr)

	A	B	C
"Top" Prefragment	40S	39Cl	42Si
"Bottom" Prefragment	39P	38S	41Al
Final Prefragment mass	39.0	38.2	41.1
Energy excitation (MeV)	112.7	125.2	83.2
Probability	1.70e-04	1.88e-04	4.26e-03
Corrected Probability		3.17e-03	1.11e-02

Starting v.11.1.102
"C1" is recommended option!

OT@MSU 09/11/2019

7

Figure 6. Example from the 2019 revision: for 48Ca at 140 MeV/u producing 32Mg, C1 is recommended starting with v11.1.102.

6.5 Practical algorithm for the initial prefragment search

A practical way to describe the LISE^{++} algorithm is:

1. For the requested final isotope, build a set of possible parent/prefragment candidates that can evaporate into the final residue.
2. For each candidate, calculate or estimate the abrasion probability, including the number of removed nucleons and the probability of the proton/neutron composition.

3. Assign an excitation-energy scale to the candidate, using either the historical surface/dSurface choice or the updated E* per abraded nucleon choice.
4. Estimate the evaporation-feeding weight W from the candidate prefragment to the final fragment.
5. Form the search probability P. In option C this is P = W · CS_geom · factorial.
6. Select the most probable prefragment, or define the top/bottom search range shown in the dialog, and pass its A_PF and E_S to the Convolution momentum model.
7. Calculate τ, σ_PF, the convolution profile, and then transform the momentum distribution into the observable Bρ or energy domain used for transmission.

Why C1 matters
C1 makes the momentum-distribution calculation more consistent with the LISE^{++} abrasion-ablation physics. It does not only change a GUI default; it changes which parent/prefragment is selected and therefore changes A_PF, E_S, τ, the peak shift, and the accepted fraction through the separator.

7. Implementation workflow in LISE^{++}

The Convolution method is not an isolated formula. In LISE^{++} it operates inside a larger calculation chain. The following workflow summarizes how the model is used for a fragment-transmission calculation.

Step	LISE^{++} operation	Physics / numerical role
1	Choose projectile, target, beam energy, production mechanism, and final isotope	Defines A_P, Z_P, target energy loss, and the isotope to be transported
2	Evaluate production cross section	The absolute yield may come from EPAX, AA, user CS, or other models; the Convolution model mainly controls the momentum shape
3	Search for initial prefragment	Determines A_PF, E*, possible evaporation path, and E_S
4	Choose separation-energy model	Mass difference, surface excess, sum, or revised method used by the selected Convolution mode
5	Compute σ_PF and τ	σ_PF defines the Gaussian core; τ defines the dissipative exponential tail
6	Build longitudinal momentum distribution	Use the Gaussian-exponential convolution with the error-function term
7	Apply target/degrader energy loss and separator optics	Convert momentum/energy profile to Bρ acceptance and calculate transmission
8	Report yield/rate/transmission	Accepted fraction depends strongly on centroid, width, and low-momentum tail

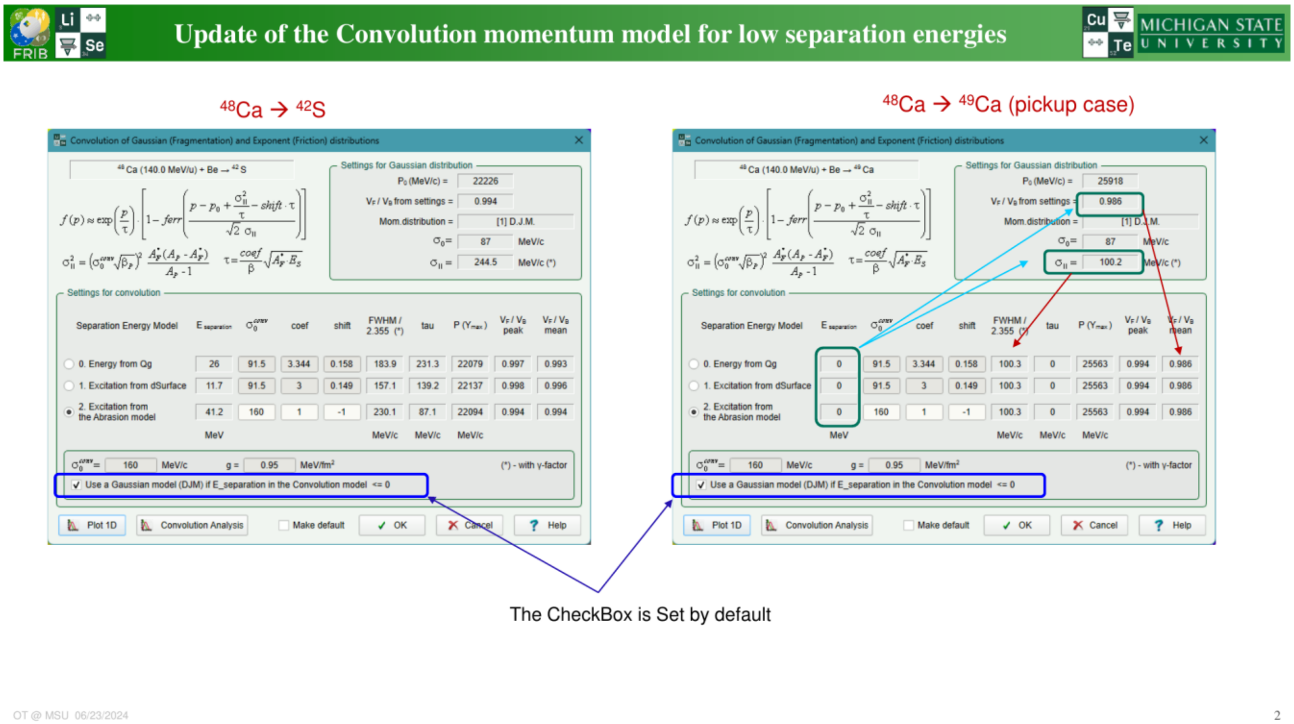
Because separator transmission is often a steep function of Bρ and momentum width, even modest differences in the predicted distribution can affect calculated rates. This is especially true for low- and intermediate-energy fragmentation, heavy targets, and very exotic residues where the momentum tail may overlap or miss the separator acceptance.

8. Low-separation-energy, pickup, and high-energy corrections

The Universal parameterization has been maintained and corrected in later LISE^{++} versions. The most relevant corrections for documentation are related to low separation energy, pickup-like edge cases, and high-energy behavior of method #2.

8.1 v17.6.11 low-separation-energy update

Version 17.6.11 introduced an update of the Convolution momentum model for low separation energies. The version note shows examples including 48Ca → 42S and a pickup case, 48Ca → 49Ca. This is important because the friction scale τ depends on E_S; when E_S is very small, a naive formula may produce unphysical shapes or numerical instabilities.



Update of the Convolution momentum model for low separation energies

^{48}Ca → ^{42}S

Convolution of Gaussian (Fragmentation) and Exponent (Friction) distributions

^{48}Ca (140.0 MeV/u) + Be → ^{42}S

$$f(p) \approx \exp\left(\frac{p}{\tau}\right)\cdot\left[1 - ferr\left(\frac{p - p_0 + \frac{\sigma_{II}^2}{\tau} - shift\cdot\tau}{\sqrt{2}\,\sigma_{II}}\right)\right]$$

$$\sigma_{II}^2 = \left(\sigma_0^{conv}\sqrt{\beta_P}\right)^2 \frac{A_F^{\bullet}(A_P - A_F^{\bullet})}{A_P - 1} \qquad \tau = \frac{coef}{\beta}\sqrt{A_F^{\bullet}\cdot E_S}$$

Settings for Gaussian distribution

P_0 (MeV/c) = 22226

V_F / V_B from settings = 0.994

Mom.distribution = [1] D.J.M.

σ_0 = 87 MeV/c

σ_{II} = 244.5 MeV/c (*)

Settings for convolution

Separation Energy Model	E separation	σ_0^{conv}	coef	shift	FWHM / 2.355 (*)	tau	P (Y max)	V_F / V_B peak	V_F / V_B mean
○ 0. Energy from Qg	26	91.5	3.344	0.158	183.9	231.3	22079	0.997	0.993
○ 1. Excitation from dSurface	11.7	91.5	3	0.149	157.1	139.2	22137	0.998	0.996
● 2. Excitation from the Abrasion model	41.2	160	1	-1	230.1	87.1	22094	0.994	0.994
	MeV				MeV/c	MeV/c	MeV/c		

σ_0^{conv} = 160 MeV/c g = 0.95 MeV/fm^2 (*) - with γ-factor

☑ Use a Gaussian model (DJM) if E_separation in the Convolution model <= 0

^{48}Ca → ^{49}Ca (pickup case)

Convolution of Gaussian (Fragmentation) and Exponent (Friction) distributions

^{48}Ca (140.0 MeV/u) + Be → ^{49}Ca

$$f(p) \approx \exp\left(\frac{p}{\tau}\right)\cdot\left[1 - ferr\left(\frac{p - p_0 + \frac{\sigma_{II}^2}{\tau} - shift\cdot\tau}{\sqrt{2}\,\sigma_{II}}\right)\right]$$

$$\sigma_{II}^2 = \left(\sigma_0^{conv}\sqrt{\beta_P}\right)^2 \frac{A_F^{\bullet}(A_P - A_F^{\bullet})}{A_P - 1} \qquad \tau = \frac{coef}{\beta}\sqrt{A_F^{\bullet}\cdot E_S}$$

Settings for Gaussian distribution

P_0 (MeV/c) = 25918

V_F / V_B from settings = 0.986

Mom.distribution = [1] D.J.M.

σ_0 = 87 MeV/c

σ_{II} = 100.2 MeV/c (*)

Settings for convolution

Separation Energy Model	E separation	σ_0^{conv}	coef	shift	FWHM / 2.355 (*)	tau	P (Y max)	V_F / V_B peak	V_F / V_B mean
○ 0. Energy from Qg	0	91.5	3.344	0.158	100.3	0	25563	0.994	0.986
○ 1. Excitation from dSurface	0	91.5	3	0.149	100.3	0	25563	0.994	0.986
● 2. Excitation from the Abrasion model	0	160	1	-1	100.3	0	25563	0.994	0.986
	MeV				MeV/c	MeV/c	MeV/c		

σ_0^{conv} = 160 MeV/c g = 0.95 MeV/fm^2 (*) - with γ-factor

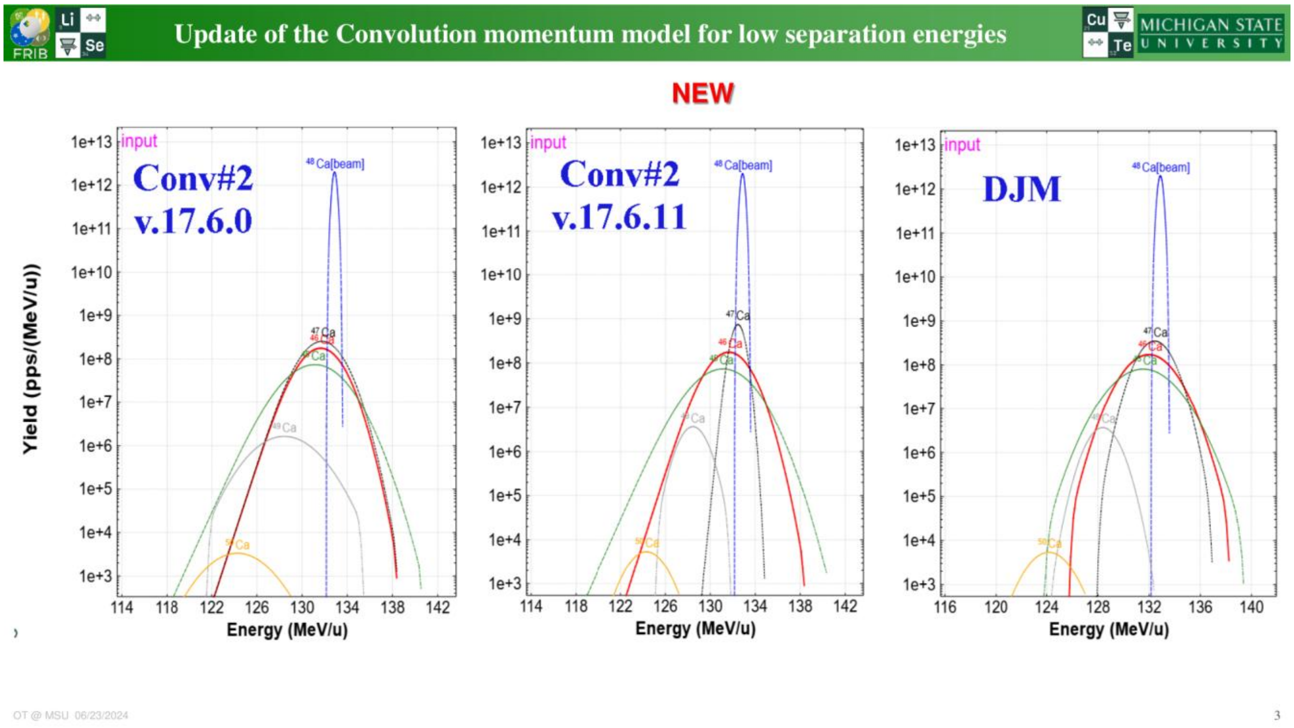
☑ Use a Gaussian model (DJM) if E_separation in the Convolution model <= 0

The CheckBox is Set by default

OT @ MSU 06/23/2024

2

Figure 7. v17.6.11 note: update of the Convolution momentum model for low separation energies, including a pickup case.



Update of the Convolution momentum model for low separation energies

NEW

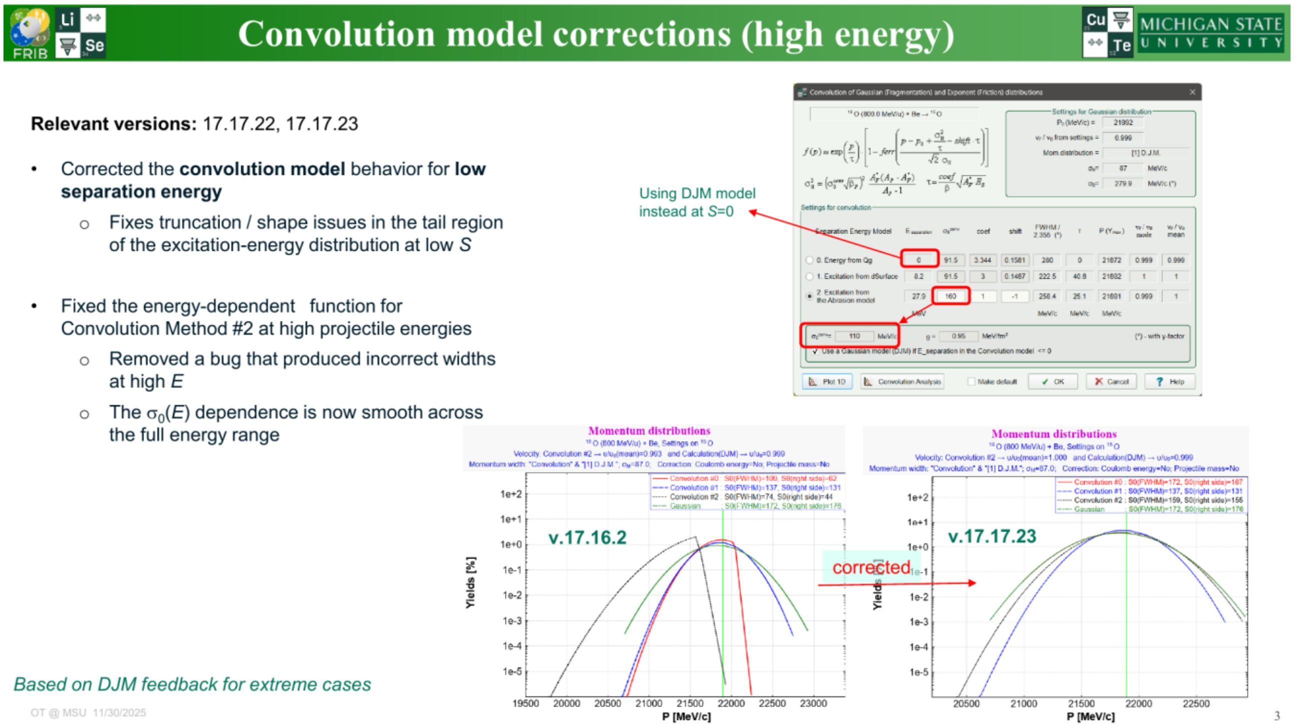
OT @ MSU 06/23/2024

3

Figure 8. v17.6.11 note showing the new low-separation-energy treatment.

8.2 v17.17.22-v17.17.23 high-energy corrections

The v17.17.29 release notes identify additional Convolution model corrections in versions 17.17.22 and 17.17.23. These include correction of the low-separation-energy behavior to avoid truncation or shape issues in the tail region, and correction of the energy-dependent function for Convolution Method #2 at high projectile energies. The latter removed a bug that produced incorrect widths at high energy and made $\sigma_0(E)$ smooth across the full energy range.



Convolution model corrections (high energy)

Relevant versions: 17.17.22, 17.17.23

- Corrected the **convolution model** behavior for **low separation energy**
  - Fixes truncation / shape issues in the tail region of the excitation-energy distribution at low S
- Fixed the energy-dependent function for Convolution Method #2 at high projectile energies
  - Removed a bug that produced incorrect widths at high E
  - The $\sigma_0(E)$ dependence is now smooth across the full energy range

Using DJM model instead at S=0

Convolution of Gaussian (Fragmentation) and Exponent (Friction) distributions

^{18}O (800.0 MeV/u) + Be → ^{15}O

$$f(p) \approx \exp\left(\frac{p}{\tau}\right)\cdot\left[1-ferr\left(\frac{p-p_0+\frac{\sigma_{\parallel}^2}{\tau}-shift\cdot\tau}{\sqrt{2}\,\sigma_{\parallel}}\right)\right]$$

$$\sigma_{\parallel}^2=\left(\sigma_0^{conv}\sqrt{\beta_P}\right)^2\frac{A_F^*(A_P-A_F^*)}{A_P-1}\qquad \tau=\frac{coef}{\beta}\sqrt{A_F^*E_S}$$

Settings for Gaussian distribution

P$_0$ (MeV/c) = 21892

v_F / v_B from settings = 0.999

Mom.distribution = [1] D.J.M.

σ_0= 87 MeV/c

$\sigma_{\parallel}$= 279.9 MeV/c (*)

Settings for convolution

Separation Energy Model	E separation	σ_0^{conv}	coef	shift	FWHM / 2.355 (*)	τ	P (Y$_{max}$)	v_F / v_B mode	v_F / v_B mean
○ 0. Energy from Qg	0	91.5	3.344	0.1581	280	0	21872	0.999	0.999
○ 1. Excitation from dSurface	8.2	91.5	3	0.1487	222.5	40.8	21882	1	1
● 2. Excitation from the Abrasion model	27.9	160	1	-1	258.4	25.1	21881	0.999	1
	MeV				MeV/c	MeV/c	MeV/c		

σ_0^{conv}= 110 MeV/c g = 0.95 MeV/fm^2 (*) - with γ-factor

☑ Use a Gaussian model (DJM) if E_separation in the Convolution model <= 0

Based on DJM feedback for extreme cases

OT @ MSU 11/30/2025

3

Figure 9. v17.17.29 note: corrections to low-separation-energy behavior and high-energy behavior of Convolution Method #2.

8.3 Practical consequence of the corrections

These fixes matter most in boundary cases:

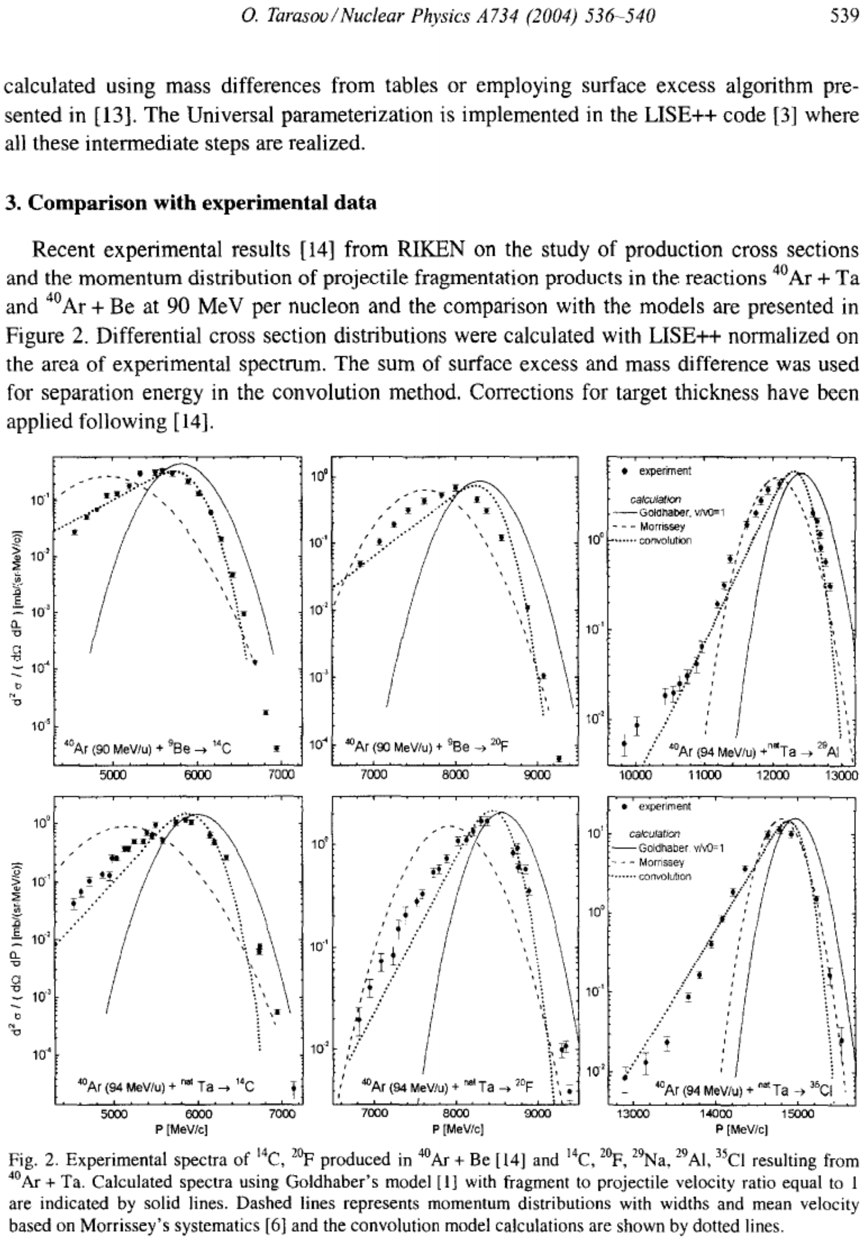
- $S \approx 0$ or very small separation energy, where τ can become too small and the error-function tail can be truncated or distorted
- Pickup-like channels, where the final fragment has more nucleons than the nominal projectile abrasion path would suggest
- High projectile energies, where the Convolution method should smoothly approach the near-Gaussian relativistic limit
- Extreme fragments far from stability, where the chosen prefragment and E_S prescription have unusually large influence on the accepted fraction

9. Practical use and diagnostics

9.1 What the model should reproduce

A successful Convolution calculation should reproduce the following simultaneously:

- The approximate Gaussian width near the high-momentum side
- The reduction of the mean fragment velocity relative to the projectile at lower energies
- The asymmetric low-momentum tail
- The energy dependence: a stronger tail and larger velocity reduction at low/intermediate energies, and a smoother Gaussian-like behavior at high energies



O. Tarasov / Nuclear Physics A734 (2004) 536–540 539

calculated using mass differences from tables or employing surface excess algorithm presented in [13]. The Universal parameterization is implemented in the LISE++ code [3] where all these intermediate steps are realized.

3. Comparison with experimental data

Recent experimental results [14] from RIKEN on the study of production cross sections and the momentum distribution of projectile fragmentation products in the reactions ^{40}Ar + Ta and ^{40}Ar + Be at 90 MeV per nucleon and the comparison with the models are presented in Figure 2. Differential cross section distributions were calculated with LISE++ normalized on the area of experimental spectrum. The sum of surface excess and mass difference was used for separation energy in the convolution method. Corrections for target thickness have been applied following [14].

Fig. 2. Experimental spectra of ^{14}C, ^{20}F produced in ^{40}Ar + Be [14] and ^{14}C, ^{20}F, ^{29}Na, ^{29}Al, ^{35}Cl resulting from ^{40}Ar + Ta. Calculated spectra using Goldhaber's model [1] with fragment to projectile velocity ratio equal to 1 are indicated by solid lines. Dashed lines represents momentum distributions with widths and mean velocity based on Morrissey's systematics [6] and the convolution model calculations are shown by dotted lines.

Figure 10. 2004 article comparison with 40Ar fragmentation data at about 90 MeV/u. Goldhaber, Morrissey, and Convolution calculations are compared with experimental momentum spectra.

9.2 What to check when results look wrong

When a calculated transmission or Bρ optimum looks suspicious, the following checks are usually more useful than changing σ_0 by hand:

- Verify the prefragment-search option. For modern calculations, C1 is normally the recommended search configuration unless historical reproduction is required
- Check the separation-energy method used by the Convolution model. The choice affects τ and therefore the tail and centroid
- Inspect whether the case is near S = 0 or includes a pickup-like channel
- Compare the Convolution distribution with a simpler DJM/Morrissey-type calculation to isolate whether the difference comes from width, centroid, or tail
- Check whether target thickness and energy-loss corrections are applied consistently, especially when comparing to measured energy spectra
- For high-energy calculations, use a LISE^{++} version with the v17.17.22-v17.17.23 corrections so that $\sigma_0(E)$ is smooth

9.3 Limitations

The Convolution method is intentionally semi-empirical. It does not separate projectile fragmentation and transfer reactions event-by-event. Instead, transfer-like contributions to the low-momentum tail are absorbed phenomenologically into the exponential attenuation. This is practical and fast for separator calculations, but it means the model should not be interpreted as a microscopic decomposition of reaction mechanisms.

Likewise, the prefragment search is an effective choice of the most probable parent. It is not a complete event-generator history. The selected prefragment is the one most relevant to the calculated residue and distribution under the chosen LISE^{++} options.

10. Recommended wording for LISE^{++} documentation

The following text can be used directly or adapted for LISE^{++} documentation:

Suggested documentation text

The Universal parameterization ("Convolution" method) calculates the longitudinal momentum distribution of projectile-fragmentation products by folding a Gaussian prefragment distribution with an exponential attenuation term. The Gaussian component represents the statistical abrasion stage, while the exponential component represents friction-like kinetic-energy loss, nucleon exchange, and conversion of projectile motion into internal excitation. The final ablation stage broadens the distribution through light-particle and gamma emission. This construction allows LISE^{++} to describe, within one model, the distribution width, the fragment-to-projectile velocity ratio, and the low-momentum tail observed at low and intermediate projectile energies. Before the distribution is calculated, LISE^{++} searches for the most probable initial prefragment that feeds the selected final fragment. The recommended modern prefragment-search option is C1, which combines the abrasion-initial-cross-section weighting with the E* per abraded nucleon excitation-energy treatment.

A shorter GUI-help version could be:

Short GUI-help wording

Convolution method: Universal parameterization for fragment momentum distributions. The model uses an abrasion Gaussian folded with an exponential friction tail and an ablation broadening step. The calculation first searches for the most probable prefragment feeding the selected final isotope; this prefragment determines A_PF, E_S, σ_PF, and τ. Use C1 for the revised prefragment search unless reproducing older calculations.

11. Sources and references

Primary sources used to prepare this document:

[1] O. Tarasov, “Analysis of momentum distributions of projectile fragmentation products,” Nuclear Physics A734 (2004) 536-540. Uploaded as up.pdf in this conversation.

[2] LISE v4.9 documentation: “Momentum distributions of fragments. Universal parametrization.” https://lise.frib.msu.edu/4/4_9/lise_4_9.htm

[3] 2006 presentation: “Projectile fragmentation aspects to produce neutron-rich isotopes,” especially slides 7-8 for the Universal parameterization and three-step Convolution model. https://lise.frib.msu.edu/paper/2006_09_acs_pf.pdf

[4] 2019 presentation: “Universal parameterization (“Convolution” method) revision,” v11.1.102. https://lise.frib.msu.edu/10_1/11_1_102_UniversalParam.pdf

[5] LISE^{++} v17.6.11 version note: update of the Convolution momentum model for low separation energies. https://lise.frib.msu.edu/17/17_06_11.pdf

[6] LISE^{++} v17.17.29 version note: Convolution model corrections at low separation energy and high projectile energy. https://lise.frib.msu.edu/17/17_17_29.pdf

Additional physics references cited by the original article and LISE v4.9 notes include Goldhaber (1974), Morrissey (1989), Friedman (1983), Borrel et al. (1983/1986), Rami et al. (1985), Gosset et al. (1977), and the experimental data sets used in the original 35-spectrum fit.