Update of Fusion reaction mechanism in LISE^{++}

MICHIGAN STATE UNIVERSITY

1. **Fission barrier dialog modification**

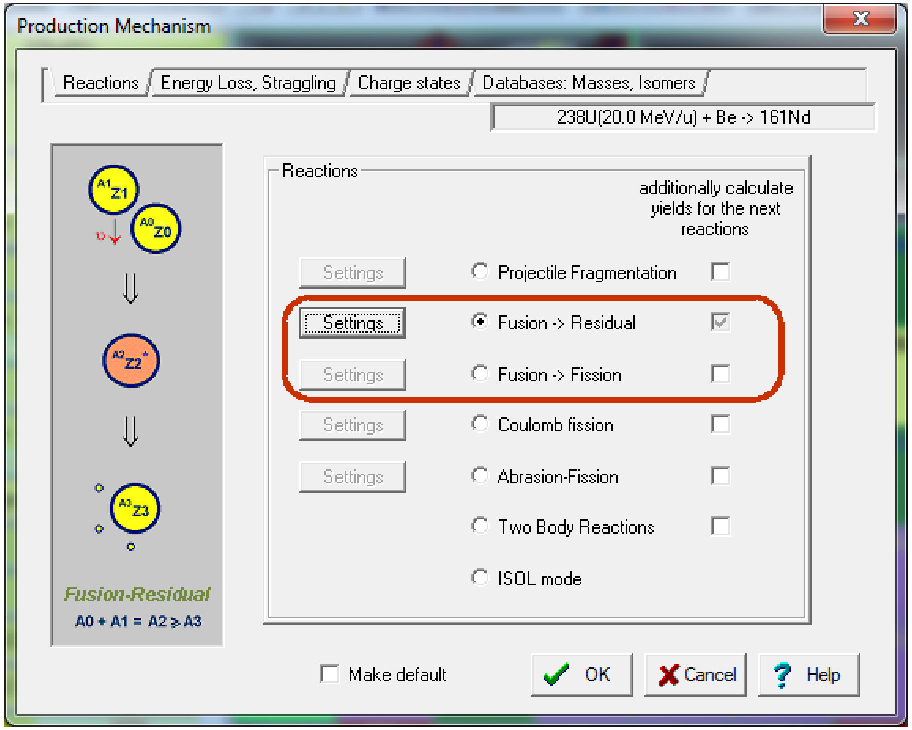
2. **Fusion dialogs**
   - *Potential energy*
   - *Potential pocket, $L_{critical}$, DIC*
   - *Fission barrier vanishing, $L_{Bfis=0}$, Fast Fission (FA)*
   - *Compound Formation, Quasi-Fission*
   - *Barrier penetration, Quasi-Elastic (QE)*
   - *Compound de-excitation : Fusion – Fission or Residue?*

3. **LISE^{++} definitions and features of fusion mechanism**

4. **Examples:**
   - ^{238}U (20 MeV/u) + Be, C
   - $^{58}Ni + ^{60}Ni$
   - ^{36}S (12 MeV/u) + ^{12}C, ^{238}U
   - $^{40}Ar + ^{208}Pb$

v.9.10.60
from 04/09/15

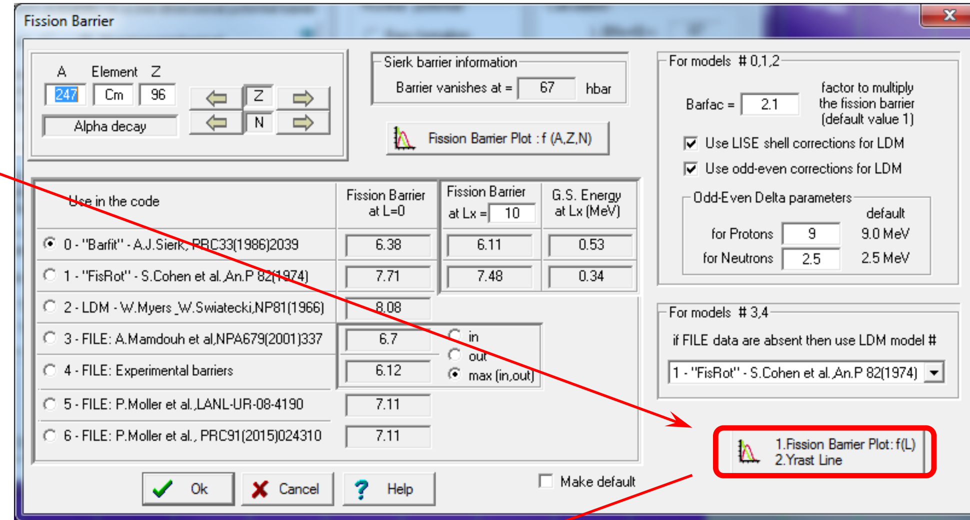
Purpose:
implementation of angular momentum formalism into low-energy reactions

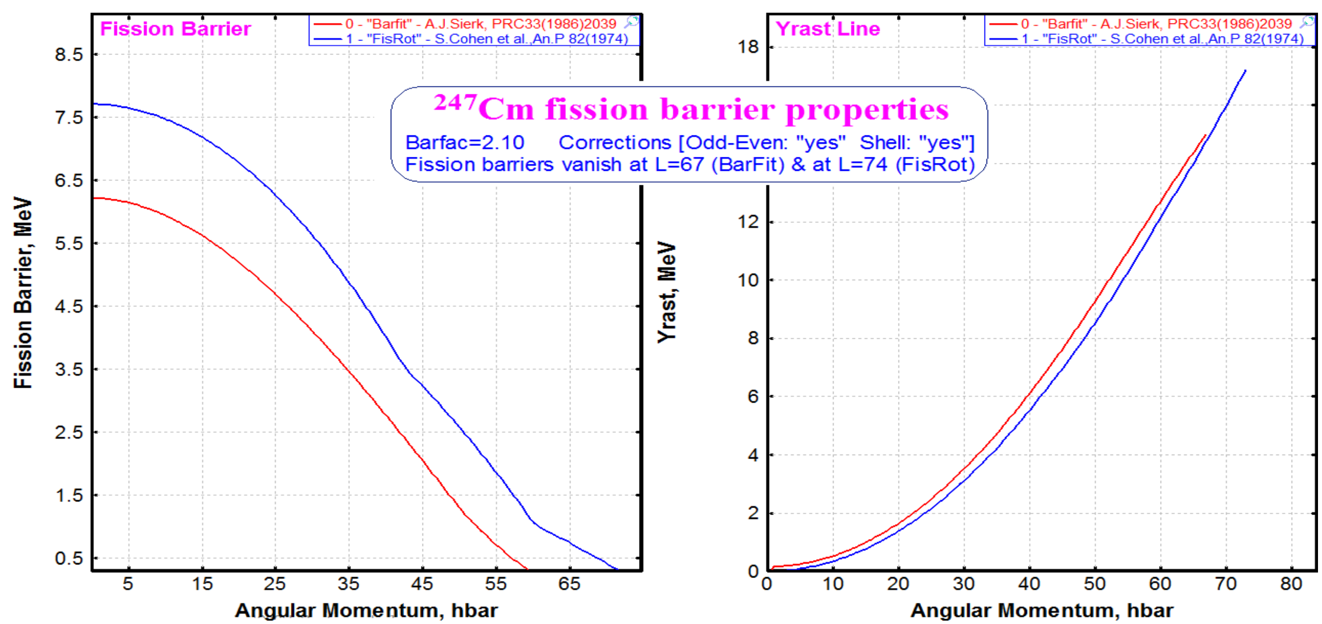


LISE^{++} documentation:
- Fusion residue transmission v.5.15
- Fusion-Fission v.7.8
- Angular momenta vs reaction channels

Fission barrier dialog modification

1. **Fission barrier and Yrast line plots as function of L**
2. New Fission barriers from P.Moller et al., PRC91(2015)024310
3. BarFac parameter for the SHE region
4. Sierk's fission barrier validity





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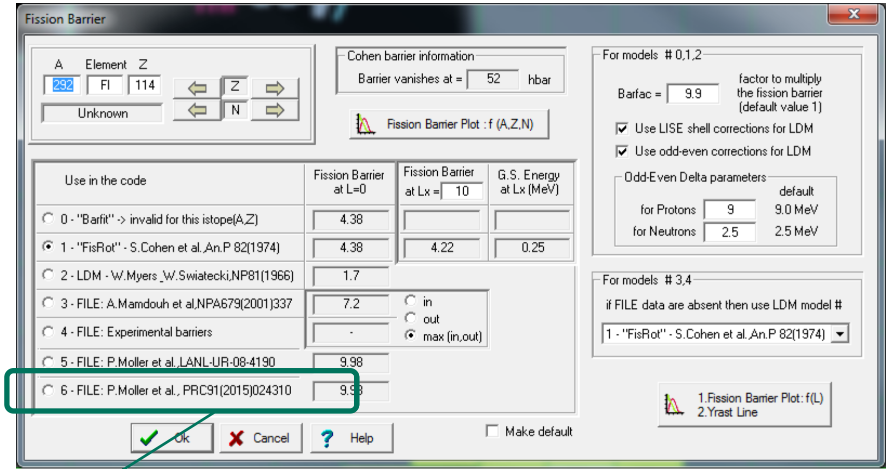
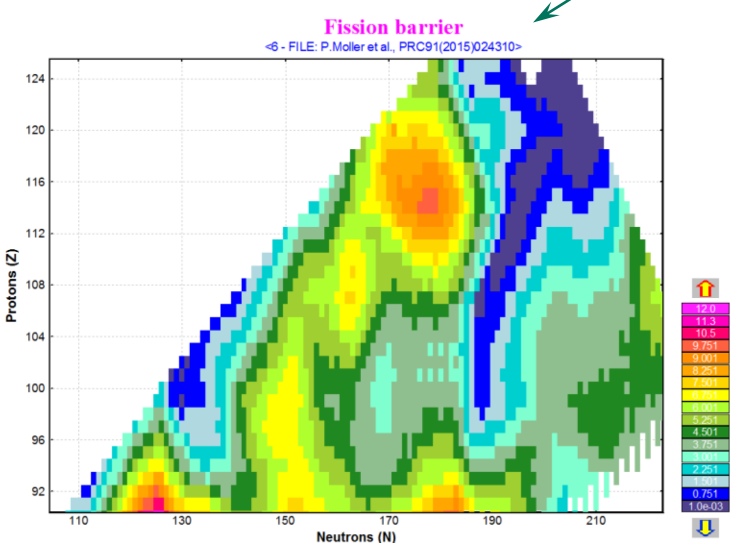


It looks like a high Barrier value with AME2012 use.

But LISE^{++} allows to load an user mass excess table



Fission barrier dialog modification

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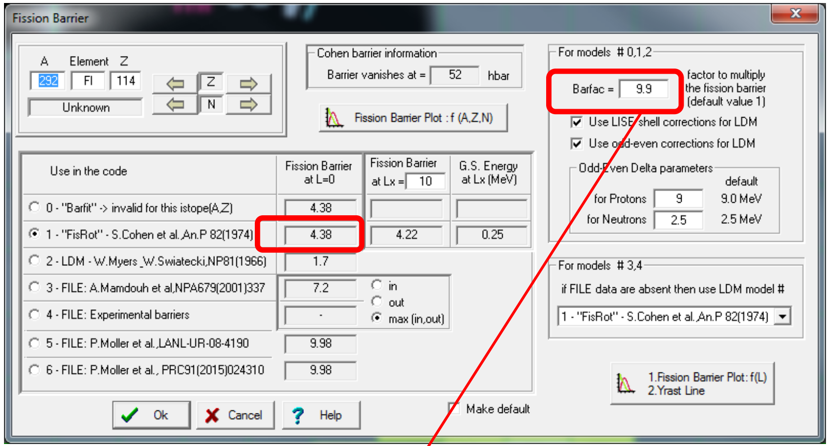
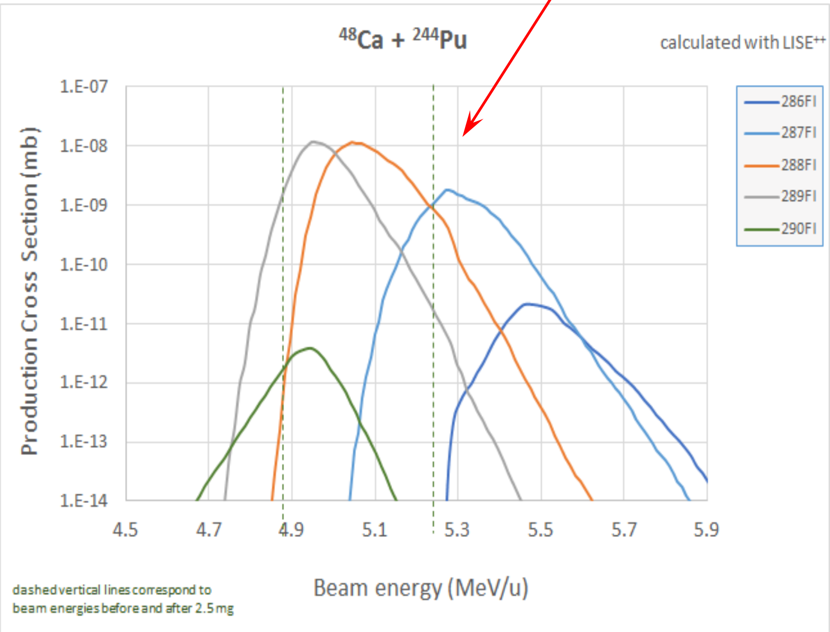


Barrier factor to describe experimental data or sophisticated calculations

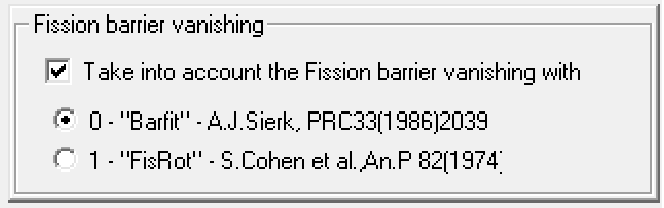
Fission barrier value plays crucial role between particle evaporation and fission competition in de-excitation process



Fission barrier dialog modification

1. Fission barrier and Yrast line plots as function of L
2. New Fission barriers from P.Moller et al., PRC91(2015)024310
3. BarFac parameter for the SHE region
4. **Sierk's fission barrier validity**

Vanishing barrier factor for data from FILEs is based on selection from Sierk or Cohen models in the Fusion dialog



0. Sierk's fission barrier is operating up to Z≤110 @ L=0 or Z ≤ 102 @ L>0

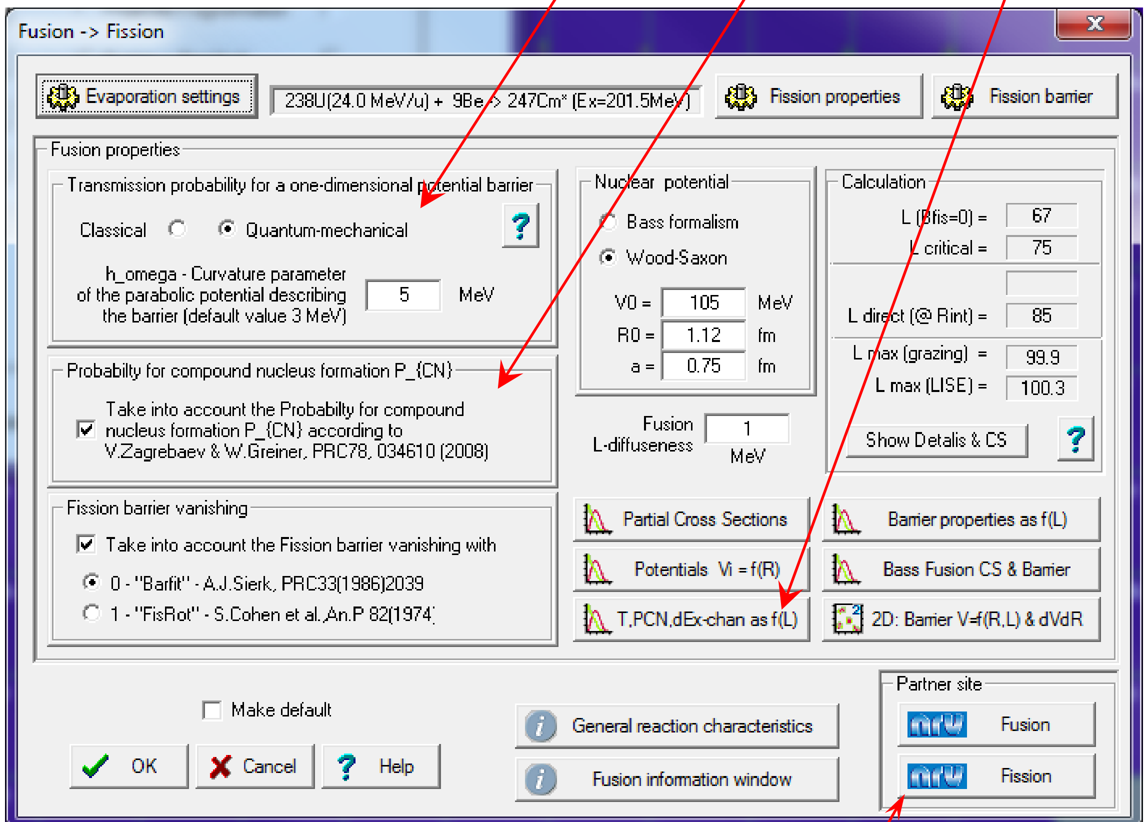
1. So, if Sierck's fission barrier has been selected , and nucleus atomic number is higher 110 at L=0, than Cohen's barrier will be used.

2. If if Sierck's fission barrier has been selected , and nucleus atomic number is higher 102 at L>0,
then vanishing factor for Z=102 (with the same N/Z ratio) will be used,

where the Vanishing factor (L) is the ratio of Barrier(A,Z,**L**) / Barrier(A,Z,**0**)

Fusion-Residue and Fusion-Fission dialogs in LISE^{++}

$$\sigma_{\rm ER}^{xn}(E) = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)P_{\rm cont}(E,l)P_{\rm CN}(E^*,l)\,P_{xn}(E^*,l)$$



Do not hesitate to use Low-Energy reaction computing centers as NRV for more sophisticated solutions with Channel Coupling, Langevin equations and so on

BJORHOLM

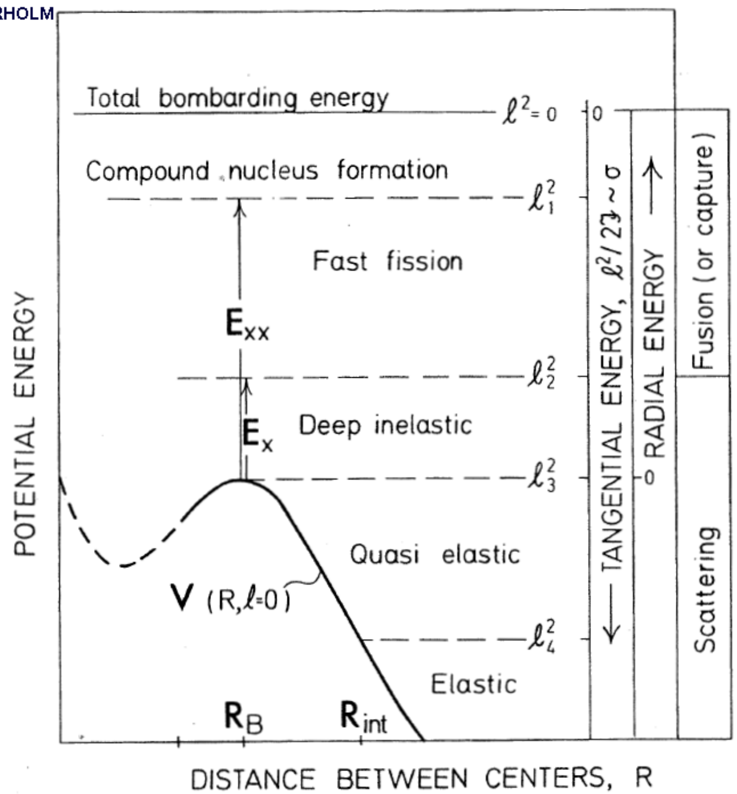


Fig. 5. Summary of the different reactions encountered in collisions between very heavy ions

C.Gregoire et al.

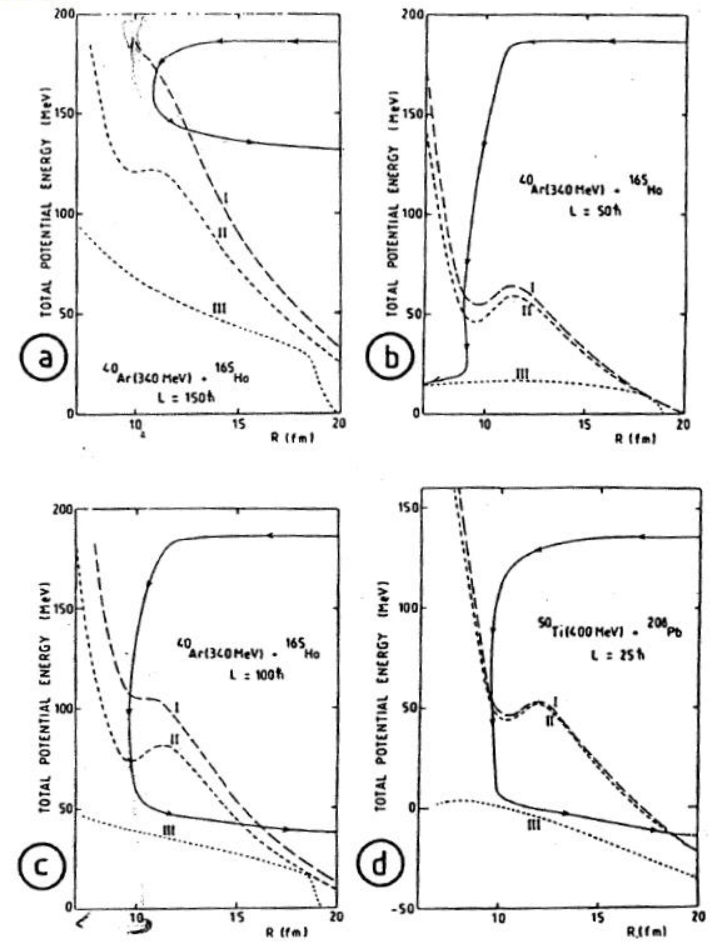
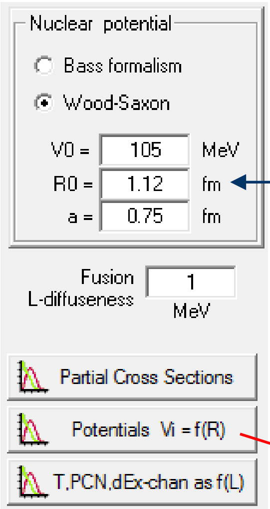


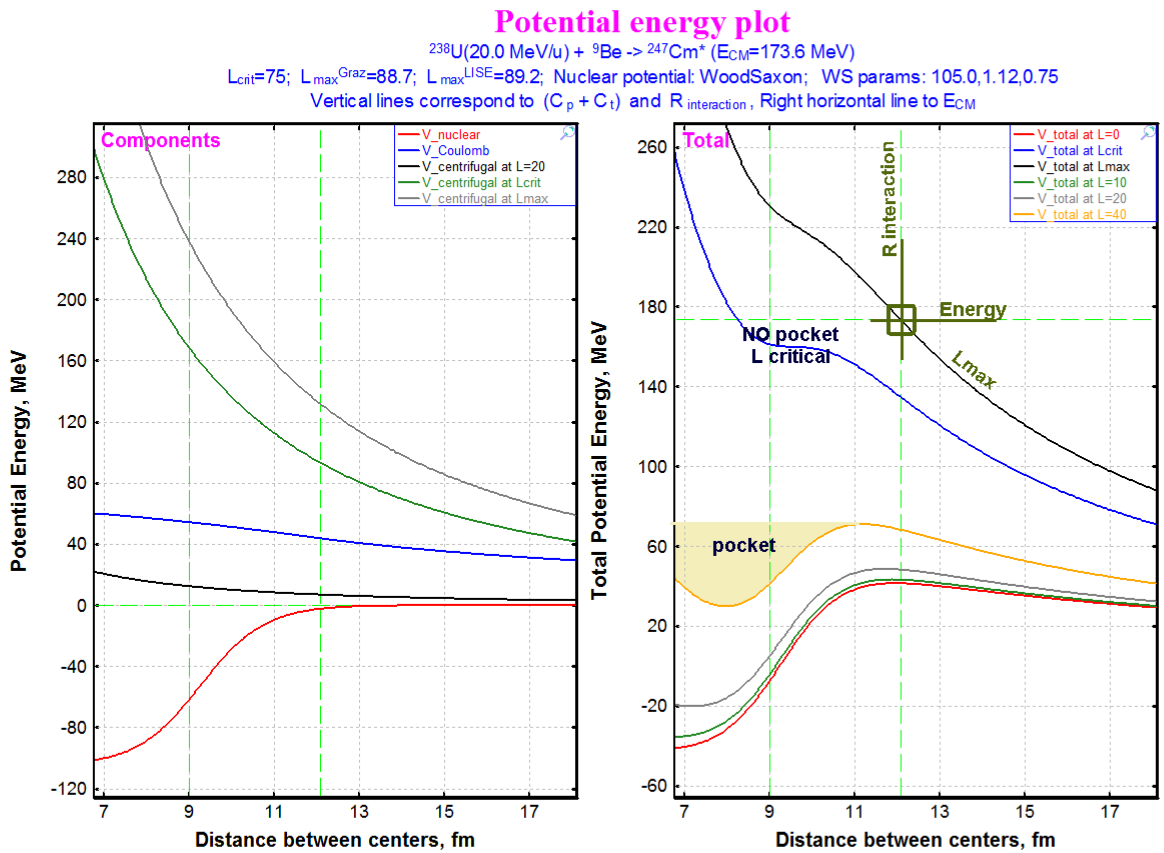
Fig. 3 - Dissipative processes in heavy ion reactions :
a) deep inelastic collisions,
b) compound nucleus formation,
c) fast fission,
d) fast fission with very heavy systems or quasi fission.

$$V(l,r) = V_{\text{nucl}}(r) + V_{\text{Coul}}(r) + l(l+1)\hbar^2/2\mu r^2$$



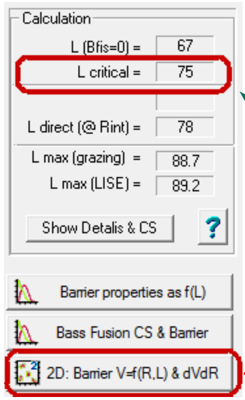
$$V_{nuc}(r) = -V_0\{1 + exp[(r - r_0(A_P^{1/3} + A_T^{1/3}))/a]\}^{-1}$$

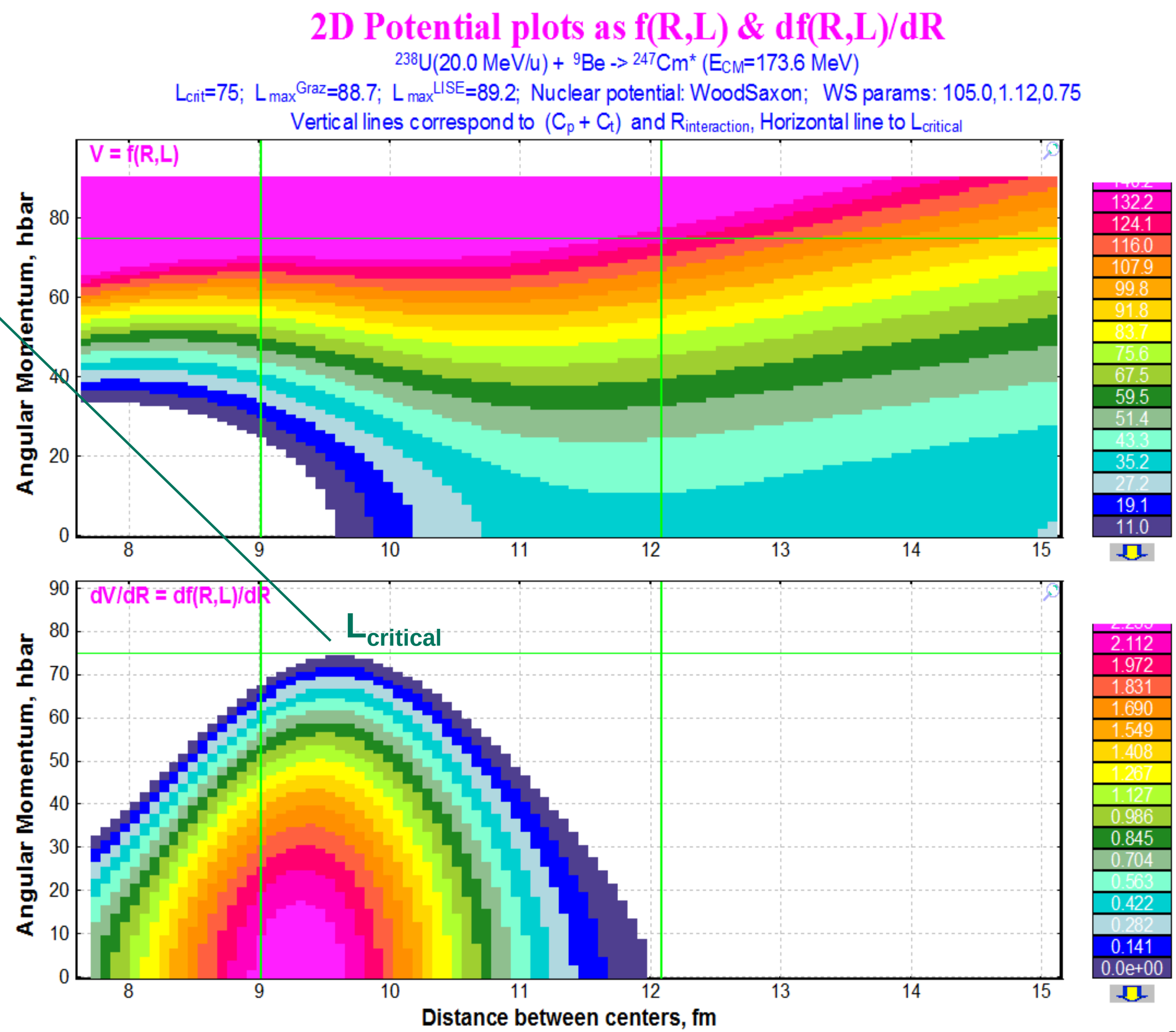
Default NRV parameters



Critical momentum

Critical momentum : L-value corresponds to potential energy when the pocket is washed out. No fusion



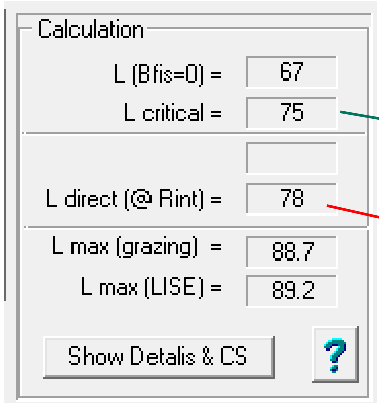


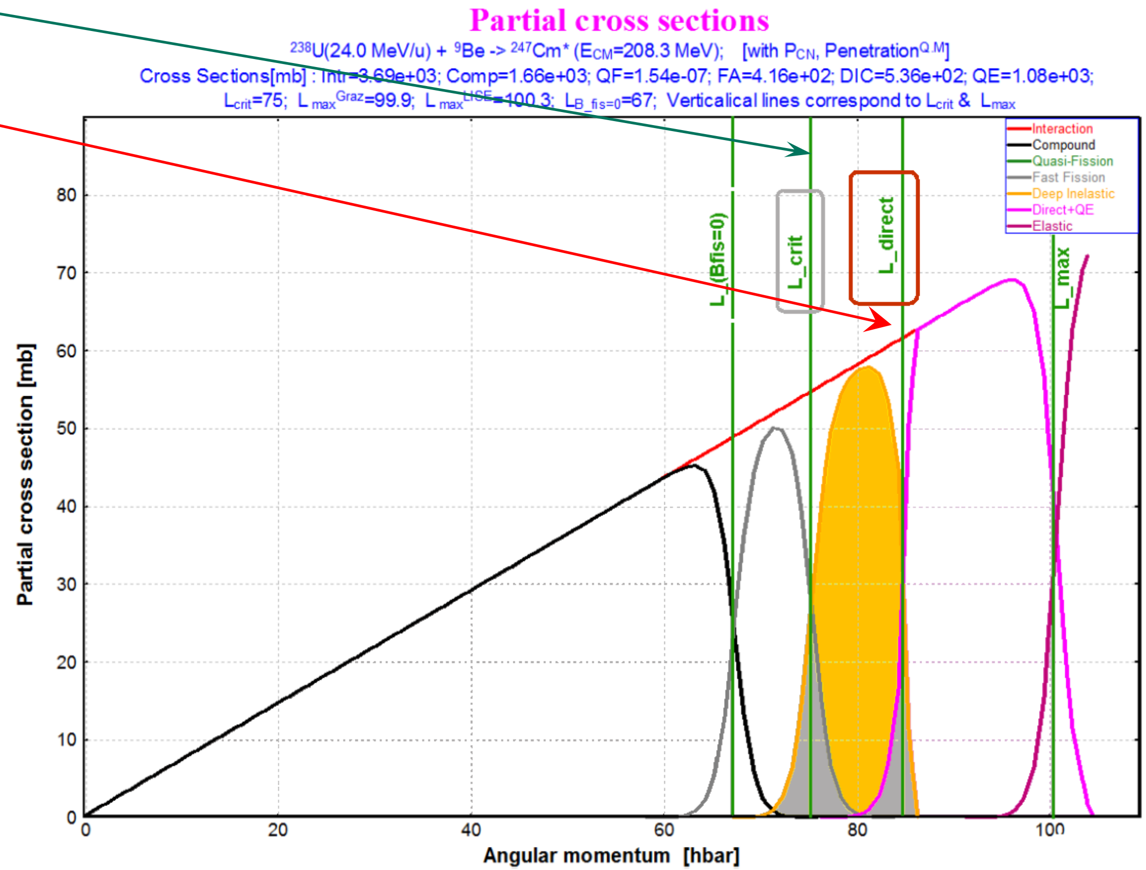
Deep Inelastic Collision region

$L_{critical} \leq L < L_{direct}$

The potential pocket does not exists, and the EnergyCM is above the barrier

Deep Inelastic Collision region



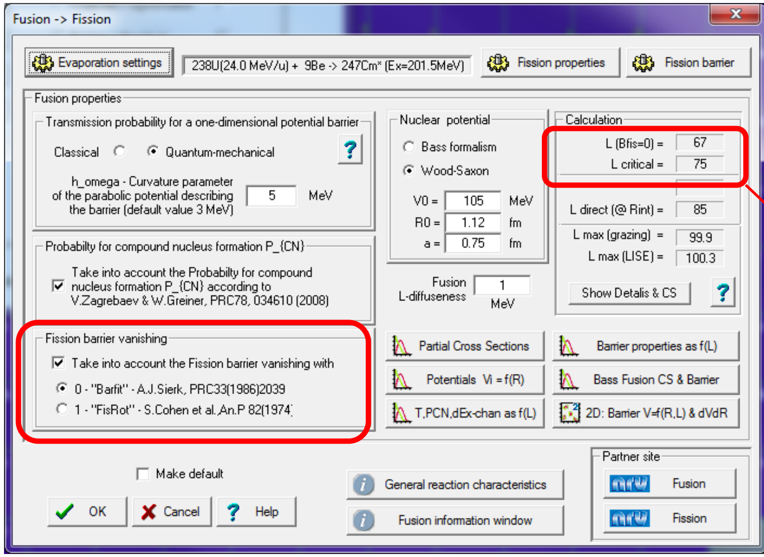


Deep Inelastic Collision region

$$L_{critical} \leq L < L_{direct}$$

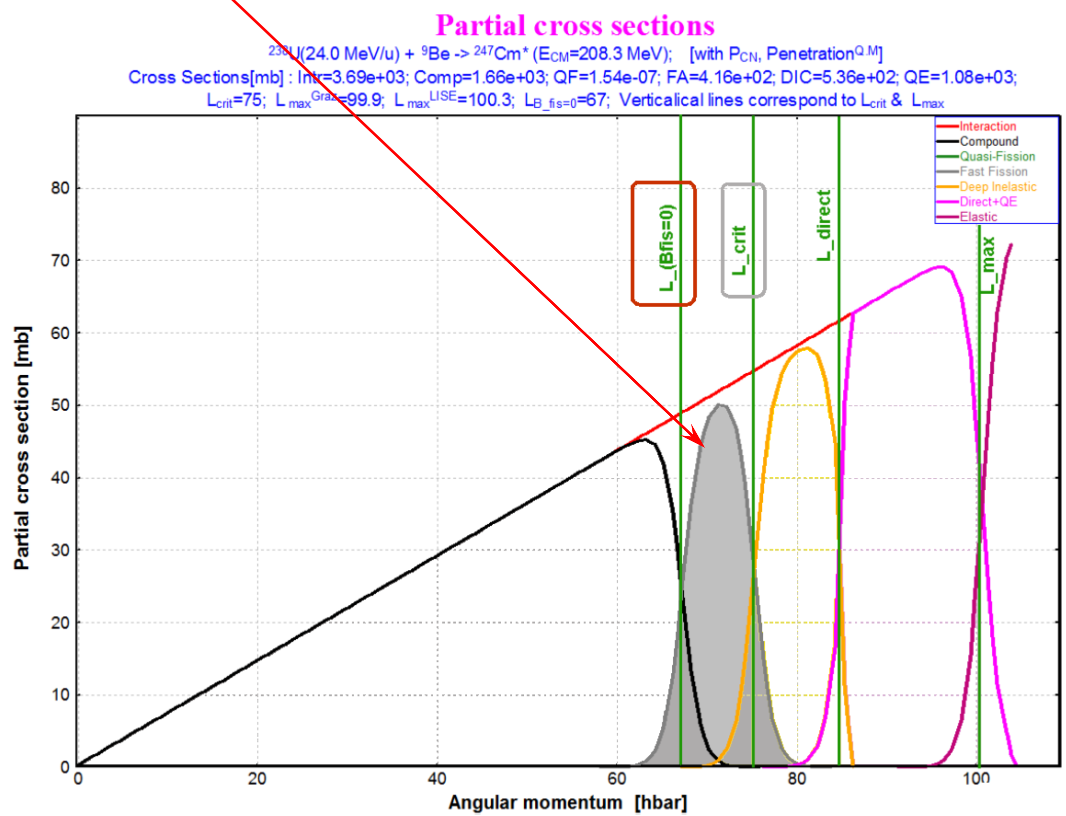
The potential pocket does not exists, and the EnergyCM is above the barrier

Fission Barrier Vanishing as f(L) → Fast Fission (FA)



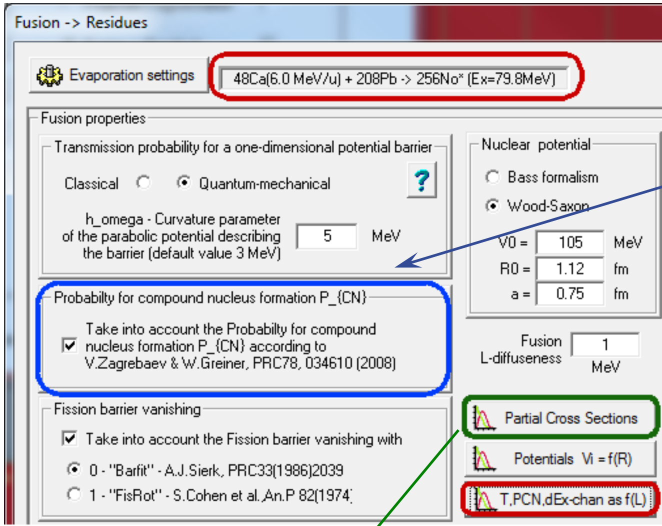
Fast Fission region

$$L_{Bfis=0} \leq L < L_{critical}$$



The potential pocket still exists, but no compound formation

Compound formation, Quasi-Fission : ^{48}Ca + ^{208}Pb



PHYSICAL REVIEW C **78**, 034610 (2008)

Synthesis of superheavy nuclei: A search for new production reactions

Valery Zagrebaev[1] and Walter Greiner[2]

$$P_{CN}(E^*, l) = \frac{P^0_{CN}}{1 + \exp\left[\frac{E^*_B - E^*_{int}(l)}{\Delta}\right]}$$

$$P^0_{CN} = \frac{1}{1 + \exp\left[\frac{Z_1 Z_2 - \zeta}{\tau}\right]}$$

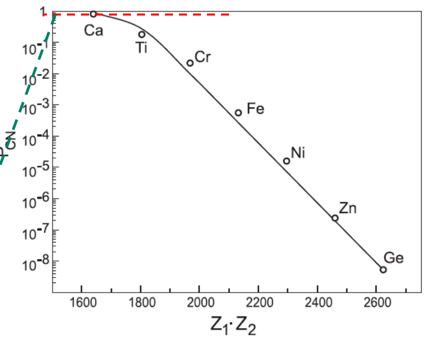
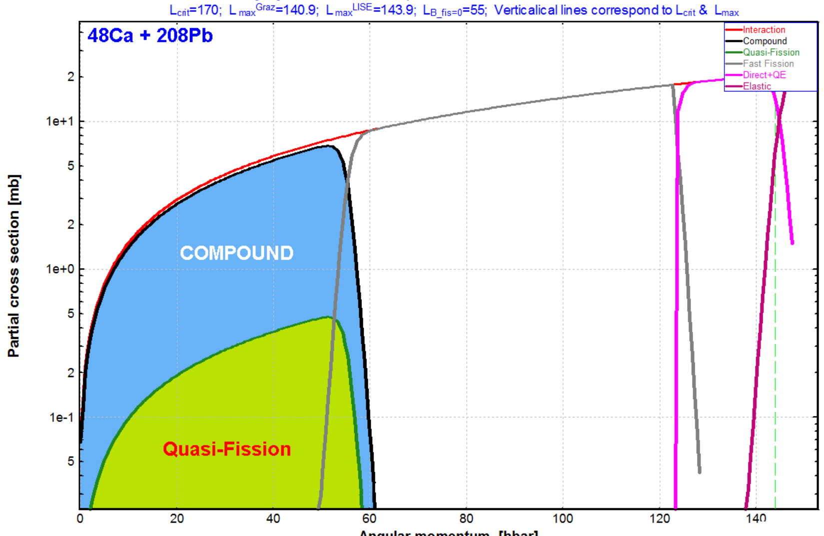


FIG. 5. Above-barrier CN formation probability in the ^{208}Pb induced fusion reactions. Results of calculation are shown by the circles, whereas the fitted curve corresponds to expression (3).

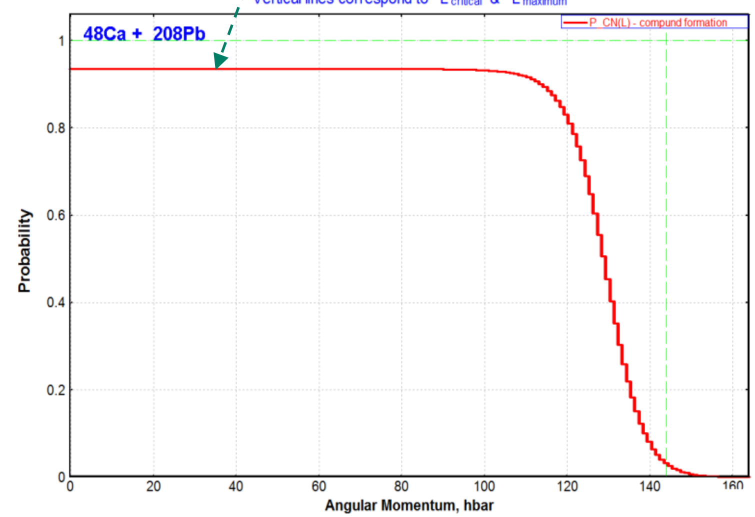
Partial cross sections

^{48}Ca(6.0 MeV/u) + ^{208}Pb -> ^{256}No* (E_{CM}=233.6 MeV); [with P_{CN}, Penetration$^{Q.M}$]
Cross Sections[mb] : Intr=1.51e+03; Comp=2.08e+02; QF=1.45e+01; FA=8.80e+02; QE=4.03e+02;
L_{crit}=170; L_{max}^{Graz}=140.9; L_{max}^{LISE}=143.9; $L_{B\_fis=0}$=55; Verticalical lines correspond to L_{crit} & L_{max}

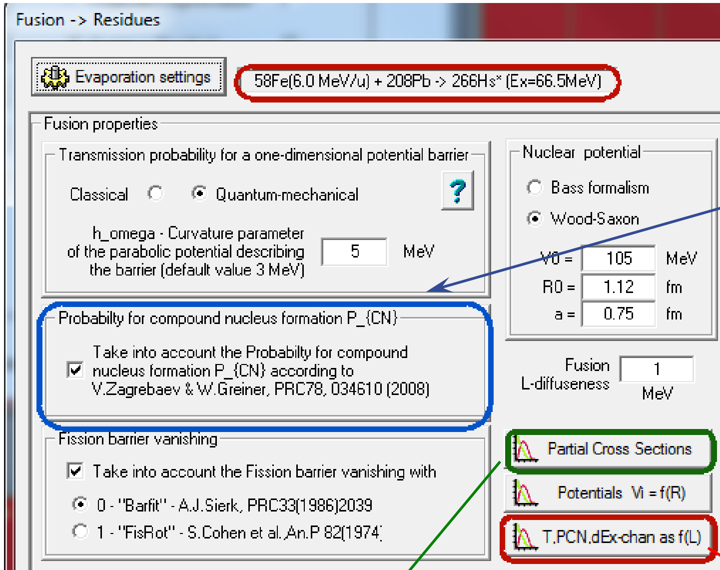


Probabilities as f (L): P_{CN}, T(L)

^{48}Ca(6.0 MeV/u) + ^{208}Pb -> ^{256}No* (E_{CM}=233.6 MeV); h_omega=5.0
L_{crit}=170; L_{max}^{Graz}=140.9; L_{max}^{LISE}=143.9; Nuclear potential: WoodSaxon
Vertical lines correspond to $L_{critical}$ & $L_{maximum}$



Compound formation, Quasi-Fission : $^{58}Fe + ^{208}Pb$



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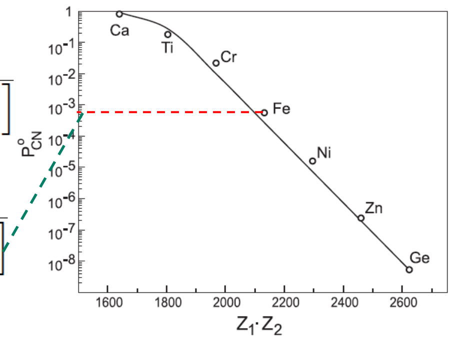

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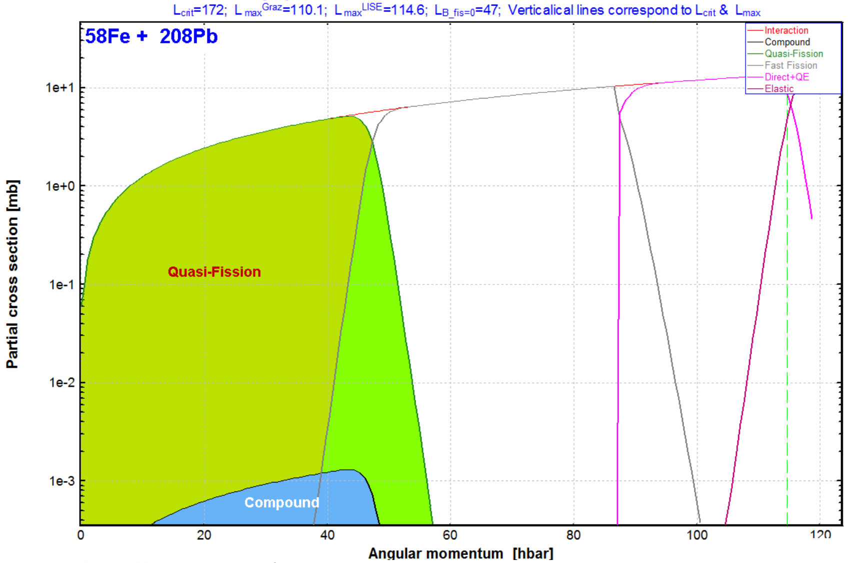
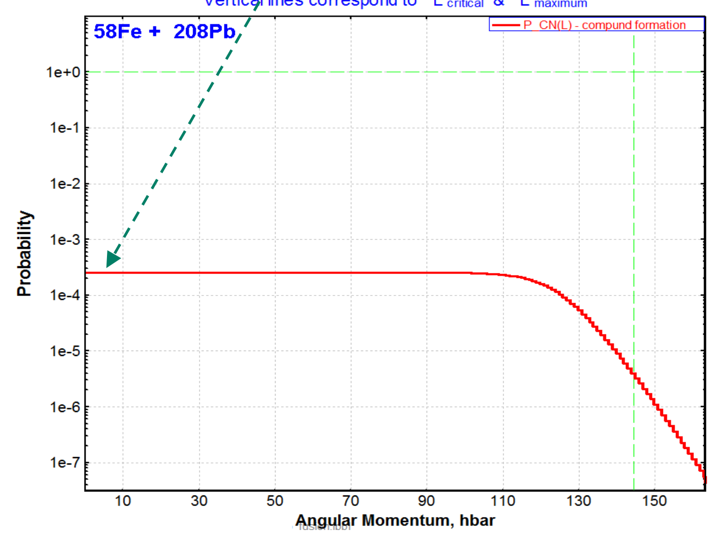
Partial cross sections

^{58}Fe(6.0 MeV/u) + ^{208}Pb -> $^{266}Hs^*$ (E_{CM}=256.7 MeV); [with P_{CN}, Penetration$^{Q.M}$]
Cross Sections[mb] : Intr=7.92e+02; Comp=3.46e-02; QF=1.35e+02; FA=3.29e+02; QE=3.28e+02;
L_{crit}=172; L_{max}^{Graz}=110.1; L_{max}^{LISE}=114.6; $L_{B\_fis=0}$=47; Verticalical lines correspond to L_{crit} & L_{max}

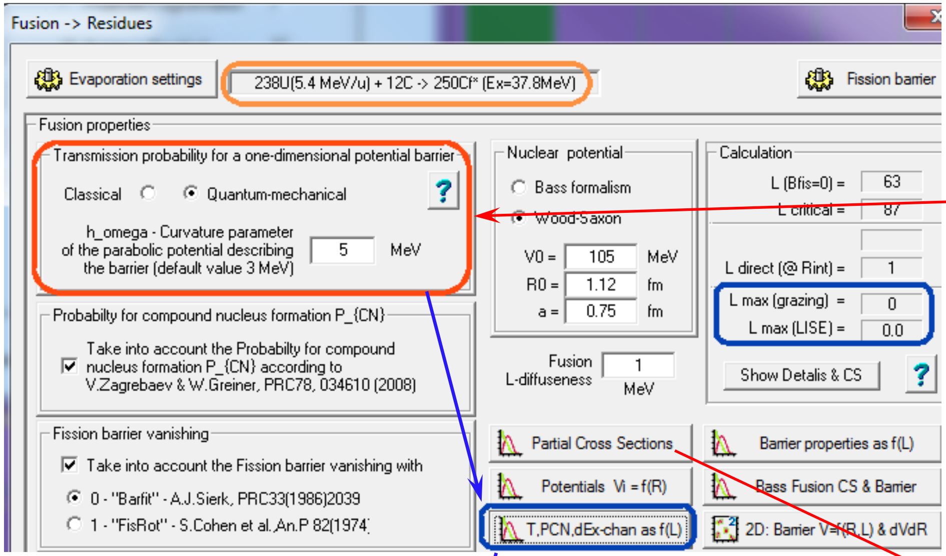


Probabilities as f (L): P_{CN}, T(L)

^{58}Fe(6.0 MeV/u) + ^{208}Pb -> $^{266}Hs^*$ (E_{CM}=271.6 MeV); h_omega=5.0
L_{crit}=180; L_{max}^{Graz}=140.6; L_{max}^{LISE}=144.4; Nuclear potential: WoodSaxon
Vertical lines correspond to $L_{critical}$ & $L_{maximum}$



Barrier penetration, Quasi-Elastic (QE) : ^{238}U(5.4 MeV/u) + C



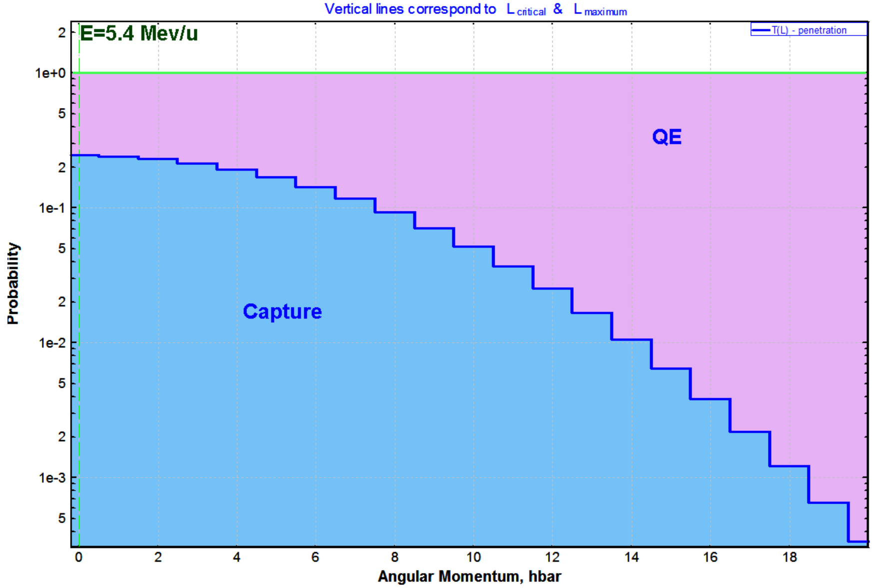
$T_l(E)$ denotes the transmission coefficient of the l-th partial wave through the barrier in the total potential

Hill–Wheeler expression

$$T_l(E) = \left(1 + \exp[2\pi(V(l,R_l) - E)/\hbar\omega]\right)^{-1}$$

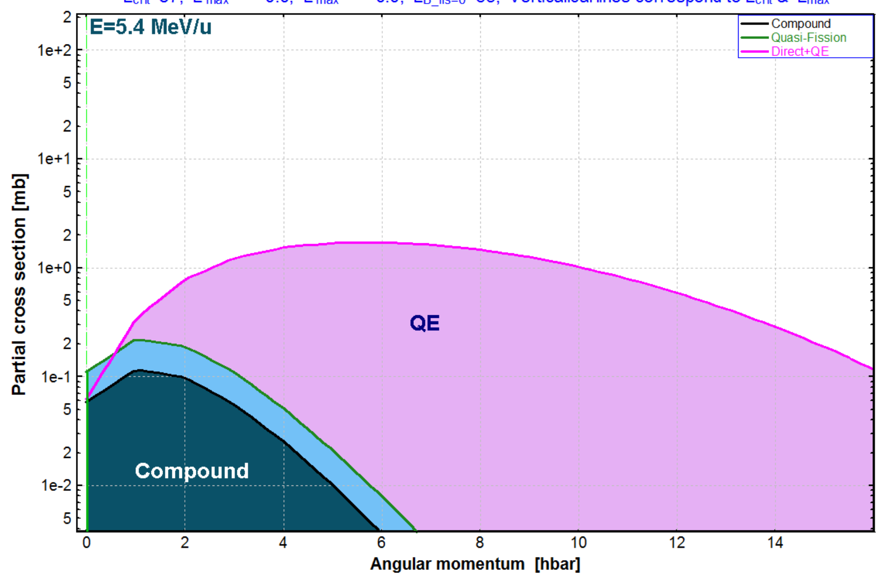
Probabilities as f (L): P_{CN}, T(L)

^{238}U(5.4 MeV/u) + ^{12}C -> ^{250}Cf* (E_{CM}=61.7 MeV); h_omega=5.0
L_{crit}=87; L_{max}^{Graz}=0.0; L_{max}^{LISE}=0.0; Nuclear potential: WoodSaxon
Vertical lines correspond to $L_{critical}$ & $L_{maximum}$

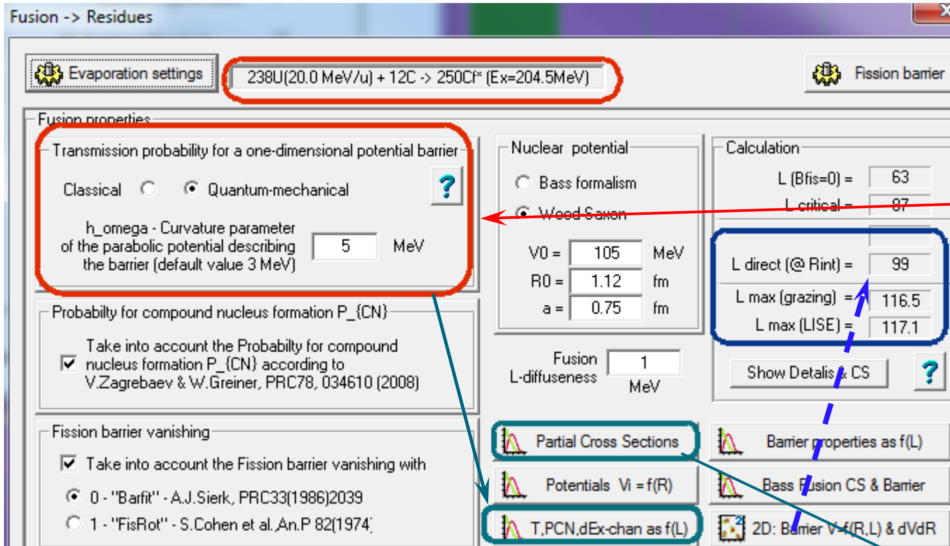


Partial cross sections

^{238}U(5.4 MeV/u) + ^{12}C -> ^{250}Cf* (E_{CM}=61.7 MeV); [with P_{CN}, Penetration$^{Q.M}$]
Cross Sections[mb] : Intr=1.62e+01; Comp=3.70e-01; QF=7.16e-01; QE=1.51e+01;
L_{crit}=87; L_{max}^{Graz}=0.0; L_{max}^{LISE}=0.0; $L_{B\_fis=0}$=63; Verticalical lines correspond to L_{crit} & L_{max}



Barrier penetration, Quasi-Elastic (QE) : ^{238}U(20.0 MeV/u) + C



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Hill–Wheeler expression

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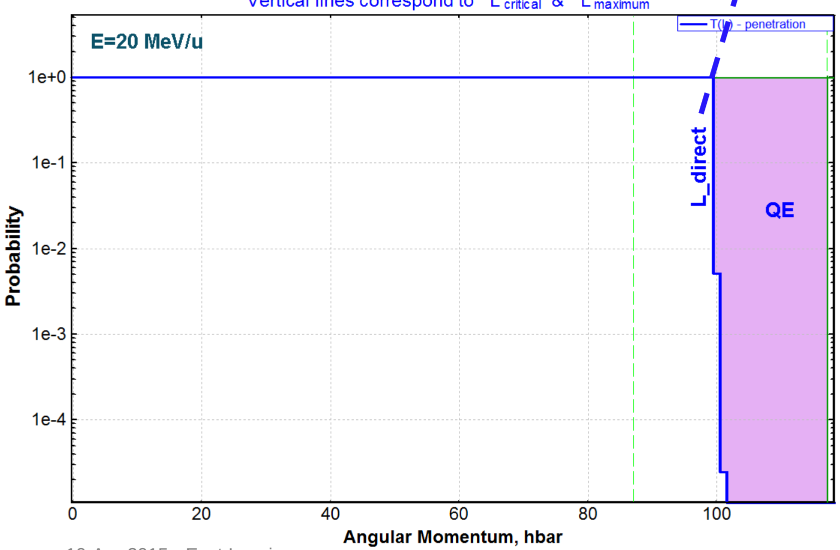
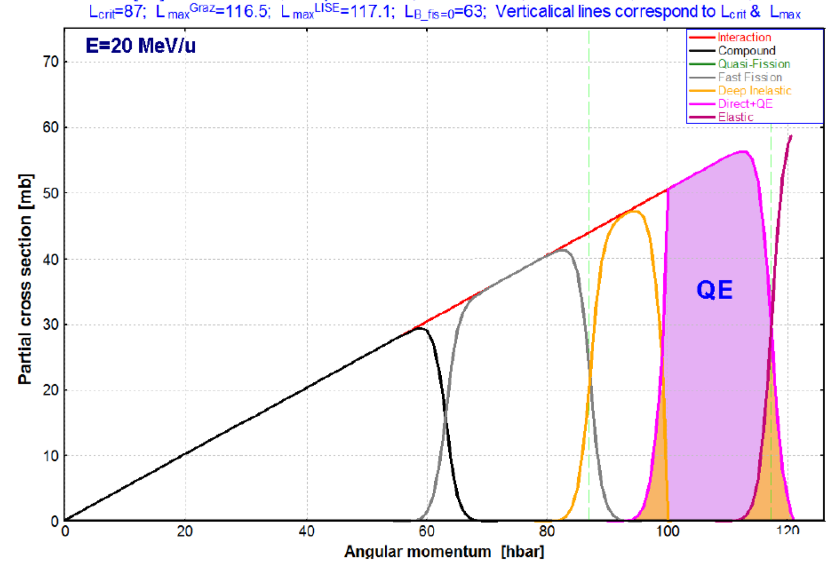
Probabilities as f (L): P_{CN}, T(L)

^{238}U(20.0 MeV/u) + ^{12}C -> ^{250}Cf* (E_{CM}=228.4 MeV); h_omega=5.0
L_{crit}=87; L_{max}^{Graz}=116.5; L_{max}^{LISE}=117.1; Nuclear potential: WoodSaxon
Vertical lines correspond to $L_{critical}$ & $L_{maximum}$

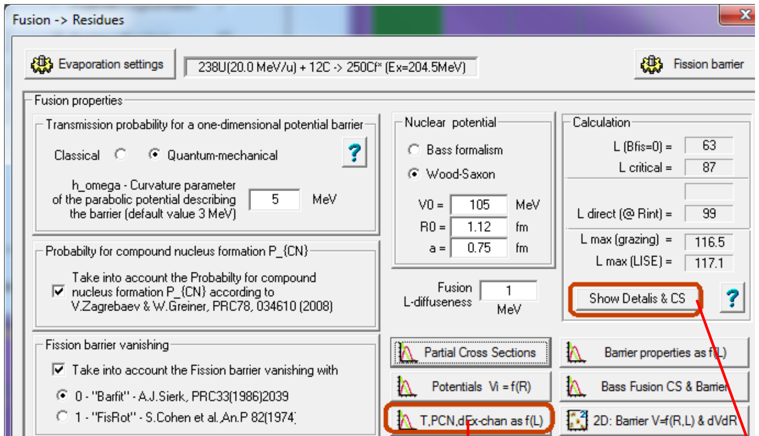


Partial cross sections

^{238}U(20.0 MeV/u) + ^{12}C -> ^{250}Cf* (E_{CM}=228.4 MeV); [with P_{CN}, Penetration^{QM}]
Cross Sections[mb] : Intr=3.48e+03; Comp=1.02e+03; QF=2.24e-09; FA=9.13e+02; DIC=5.42e+02; QE=1.01e
L_{crit}=87; L_{max}^{Graz}=116.5; L_{max}^{LISE}=117.1; $L_{B\_fis=0}$=63; Verticalical lines correspond to L_{crit} & L_{max}



1st step compound de-excitation plot

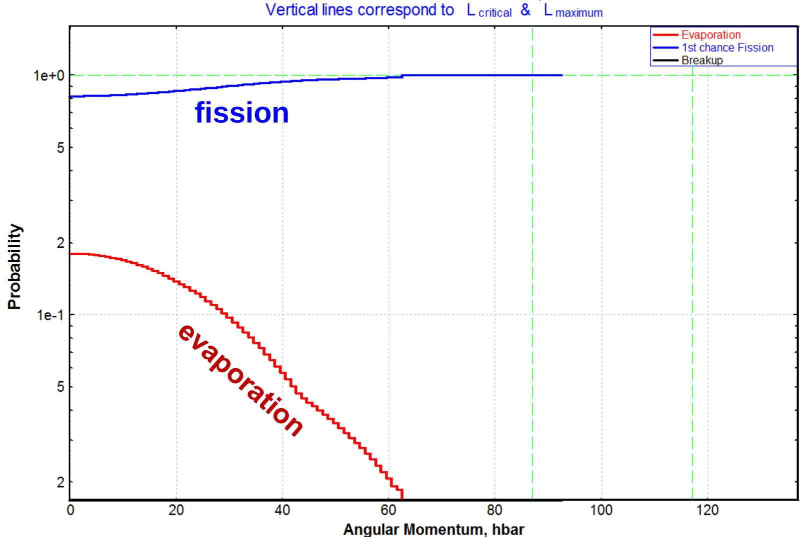


Probabilities as f (L): Compound 1st step de-exctitation

^{238}U(20.0 MeV/u) + ^{12}C -> ^{250}Cf* (E_{CM}=228.4 MeV); h_omega=5.0

L_{crit}=87; L_{max}^{Graz}=116.5; L_{max}^{LISE}=117.1; Nuclear potential: WoodSaxon

Vertical lines correspond to $L_{critical}$ & $L_{maximum}$

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238U(20.0 MeV/u) + 12C -> 250Cf* (Ex=204.5MeV)

 E_LAB=4760.01 MeV, E_CM=228.40 MeV

 Fusion -> Residues

Settings

Nuclear Potential                                          Wood-Saxon
WS parameters                                              105.0,1.12,0.
P_CN (probability of compound formation)                   Yes  (take in
P_CN at L=0                                                1.00e+00
Fission barrier vanishing                                  Yes  (take in
Transmssion probabilty for a 1-dimensional barrier         Quantum-Mecha
Curvature parameter of the parabolic potential             5.00
Fusion L-diffuseness                                       1.00

Momentum (hbar)

L (Bfis=0)                    63
L critical                    87          (E crit=176.9 MeV)
---------------------------------------------------------------
L direct (AA)                 105.3
L direct                      99          used in calculations
---------------------------------------------------------------
L max (grazing)               116.5
L max (LISE)                  117.1       used in calculations

Cross sections (mb)

 Partial (LISE++)
Interaction                   3.483e+03
Compound                      1.016e+03
Quasi-Fission                 2.237e-09
Fast Fission                  9.126e+02
Deep Inelastic                5.415e+02
Direct+QE                     1.013e+03

 Compound 1st step de-excitation channels (LISE++)
Fusion-Residue                6.436e+01
Fusion-Fission                9.516e+02
Fusion-Breackup               0.000e+00

 Cross section used in calculations (beginning of target)
Complete Fusion               2.101e+03
Use this factor for rates     0.484

 Bass cross section calculations
Fusion cross section          2.101e+03   mb
Fusion barrier                64.22       MeV
Fusion radius                 9.10        fm
Barrier position              11.55       fm
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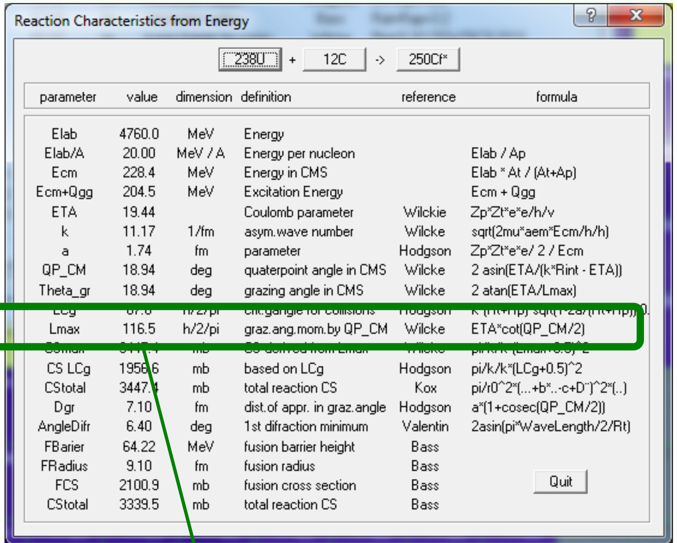
LISE++ reaction interpretation from R & L

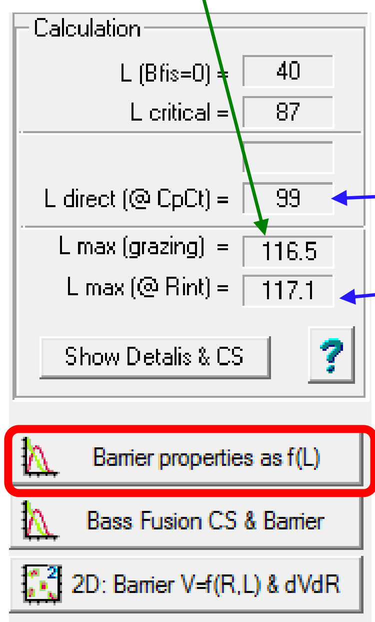


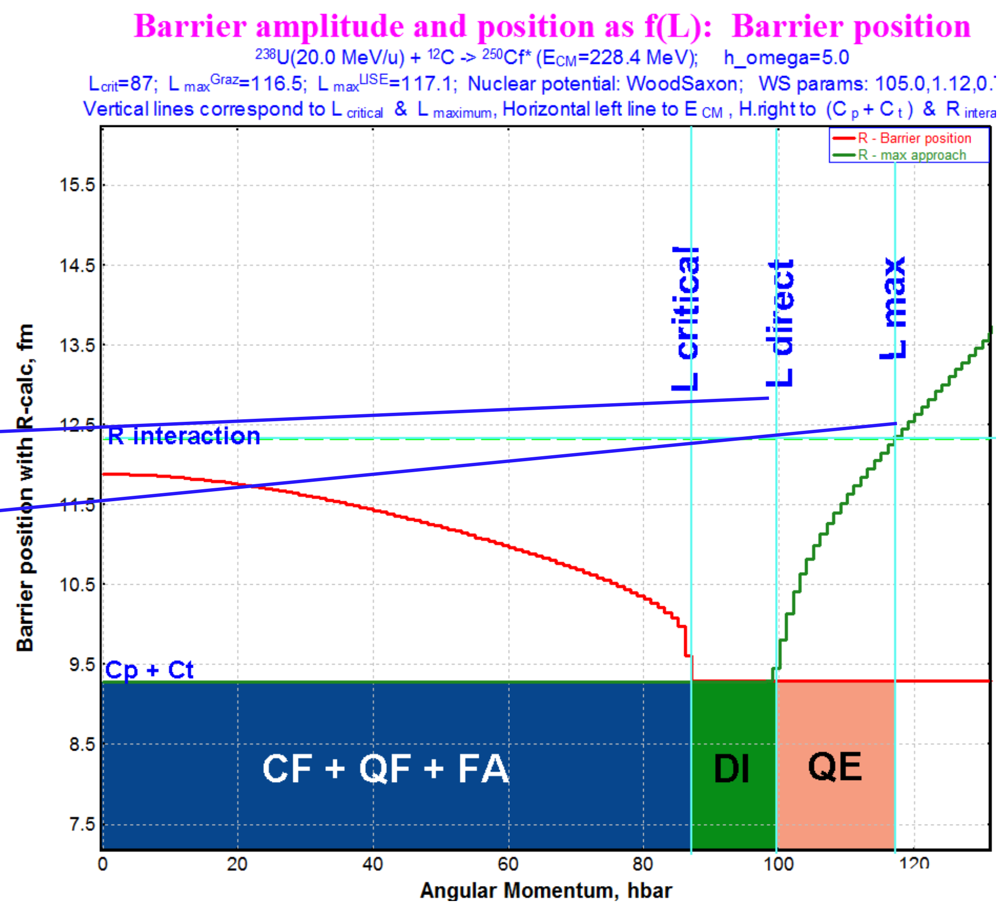
Reaction Characteristics from Energy

238U + 12C -> 250Cf*

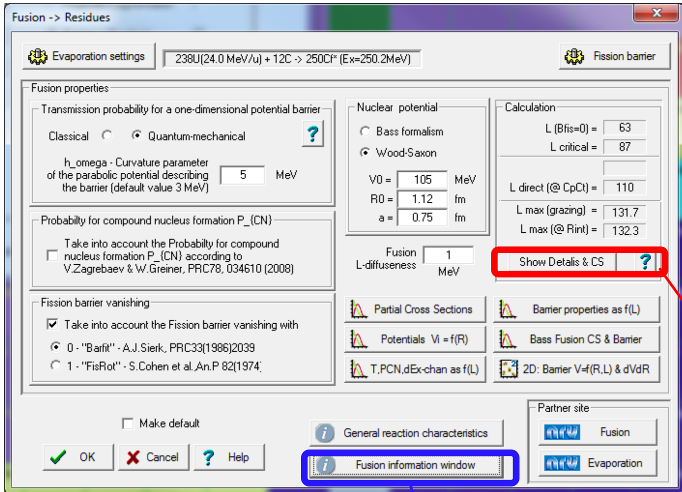
parameter	value	dimension	definition	reference	formula
Elab	4760.0	MeV	Energy		
Elab/A	20.00	MeV / A	Energy per nucleon		Elab / Ap
Ecm	228.4	MeV	Energy in CMS		Elab * At / (At+Ap)
Ecm+Qgg	204.5	MeV	Excitation Energy		Ecm + Qgg
ETA	19.44		Coulomb parameter	Wilckie	Zp*Zt*e*e/h/v
k	11.17	1/fm	asym.wave number	Wilcke	sqrt(2mu*aem*Ecm/h/h)
a	1.74	fm	parameter	Hodgson	Zp*Zt*e*e/ 2 / Ecm
QP_CM	18.94	deg	quaterpoint angle in CMS	Wilcke	2 asin(ETA/(k*Rint - ETA))
Theta_gr	18.94	deg	grazing angle in CMS	Wilcke	2 atan(ETA/Lmax)
LCg	[illegible]	h/2/pi	crit.gangle for collisions	Hodgson	[illegible]
Lmax	116.5	h/2/pi	graz.ang.mom.by QP_CM	Wilcke	ETA*cot(QP_CM/2)
CSmax	[illegible]	mb	CS derived from Lmax	Wilcke	[illegible]
CS LCg	1956.6	mb	based on LCg	Hodgson	pi/k/k*(LCg+0.5)^2
CStotal	3447.4	mb	total reaction CS	Kox	pi/r0^2*(...+b*..-c+D`)^2*(..)
Dgr	7.10	fm	dist.of appr. in graz.angle	Hodgson	a*(1+cosec(QP_CM/2))
AngleDifr	6.40	deg	1st difraction minimum	Valentin	2asin(pi*WaveLength/2/Rt)
FBarier	64.22	MeV	fusion barrier height	Bass	
FRadius	9.10	fm	fusion radius	Bass	
FCS	2100.9	mb	fusion cross section	Bass	
CStotal	3339.5	mb	total reaction CS	Bass	

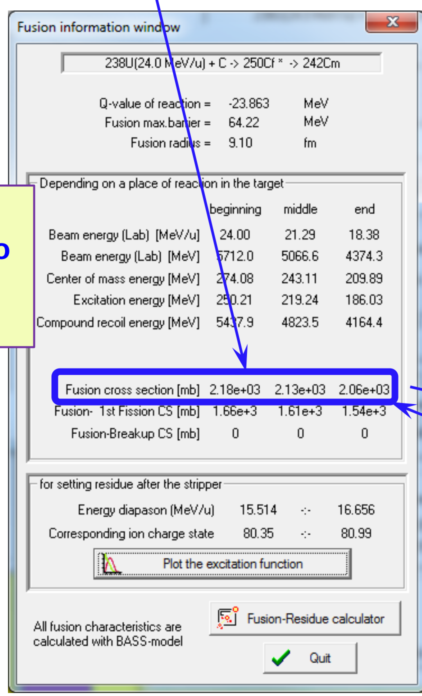
Quit

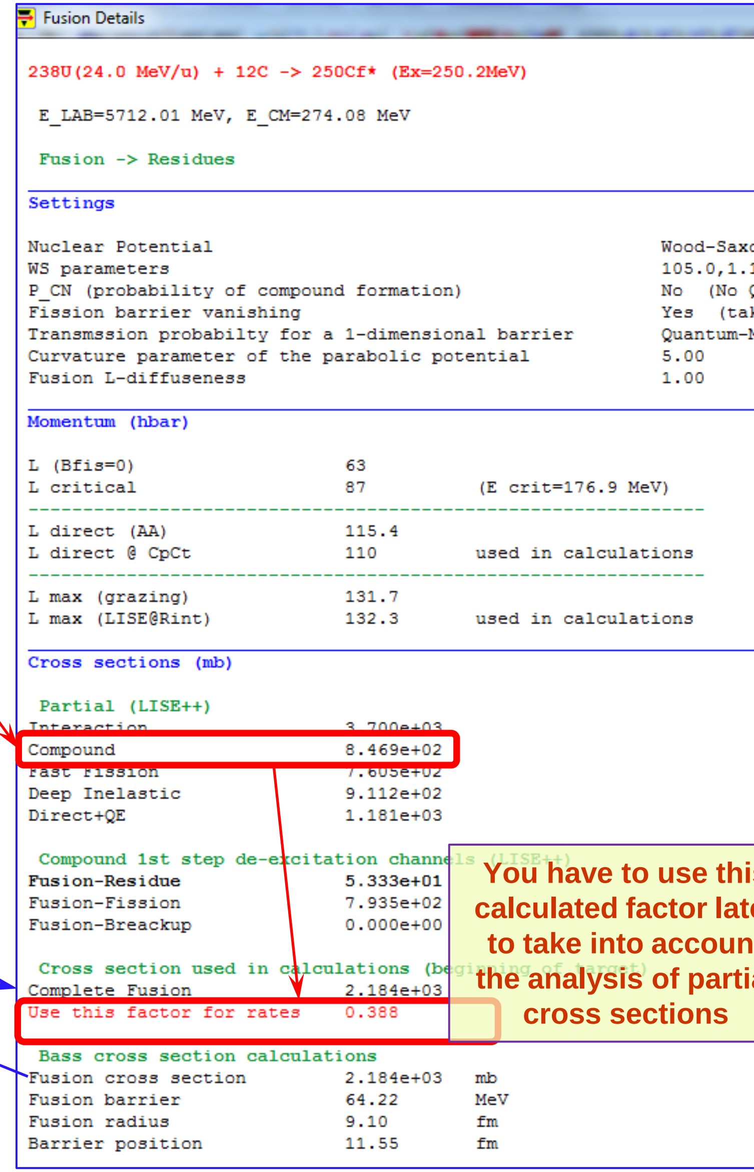




What cross sections will be used in LISE^{++} to get rates?



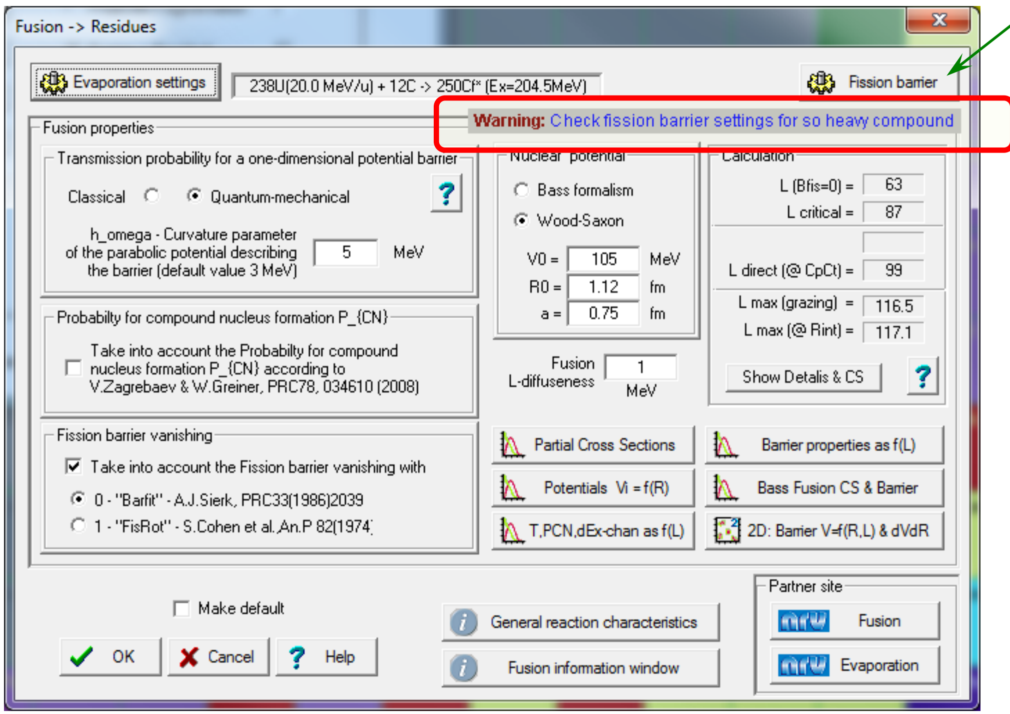




LISE^{++} still uses the Bass formalism to calculate fragment rates !!!!

You have to use this calculated factor later to take into account the analysis of partial cross sections

Fusion dialog warning message for barrier settings

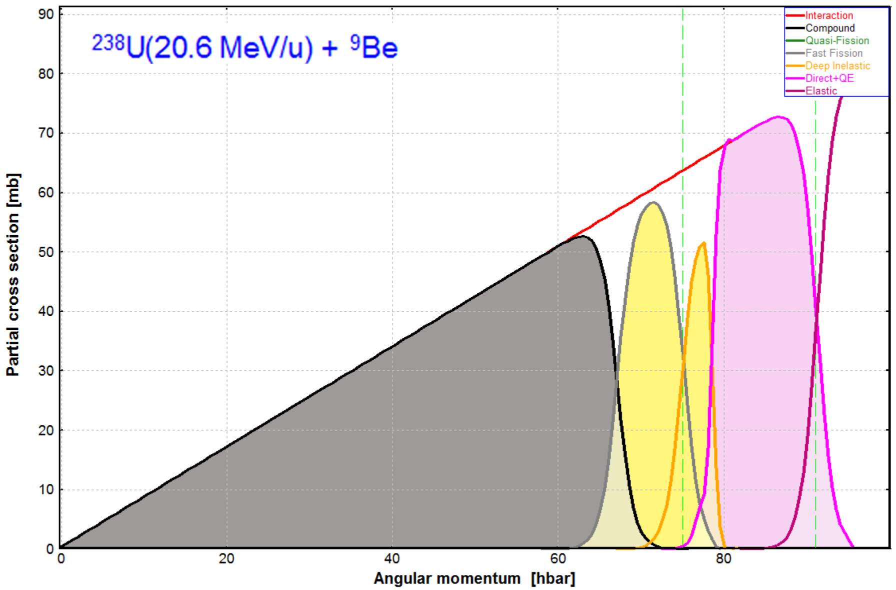


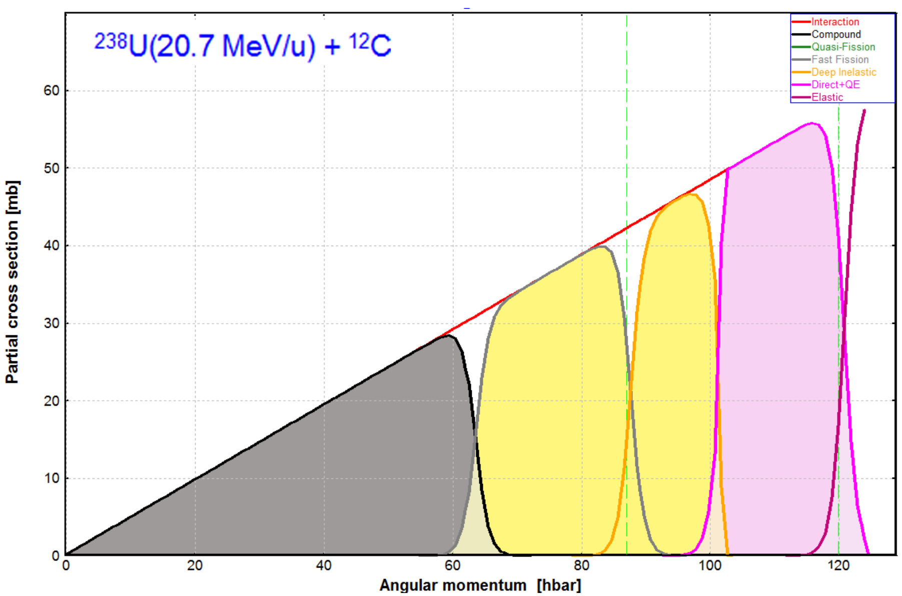
Go to the Fission barrier dialog to modify settings for this SHN region

You will get this message if

- **The compound Z > 95**
- **In-built fission model (Barfit, FisRot, or LDM) was selected**
- **“BarFac” parameter < 1.2**

^{238}U (20.6 MeV/u) with Be vs. C targets

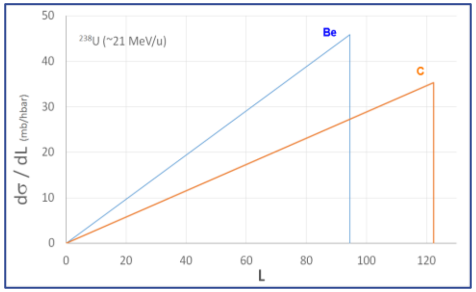




average for 17-24 MeV/u range

Fission Barrier Vanishing	Reactions	Targets: Be	Targets: C
Sierk	DIC+FA	19%	42%
	Fusion-Fission	56%	29%
	QE	25%	29%
Cohen	DIC+FA	8%	29%
	Fusion-Fission	66%	41%
	QE	25%	29%

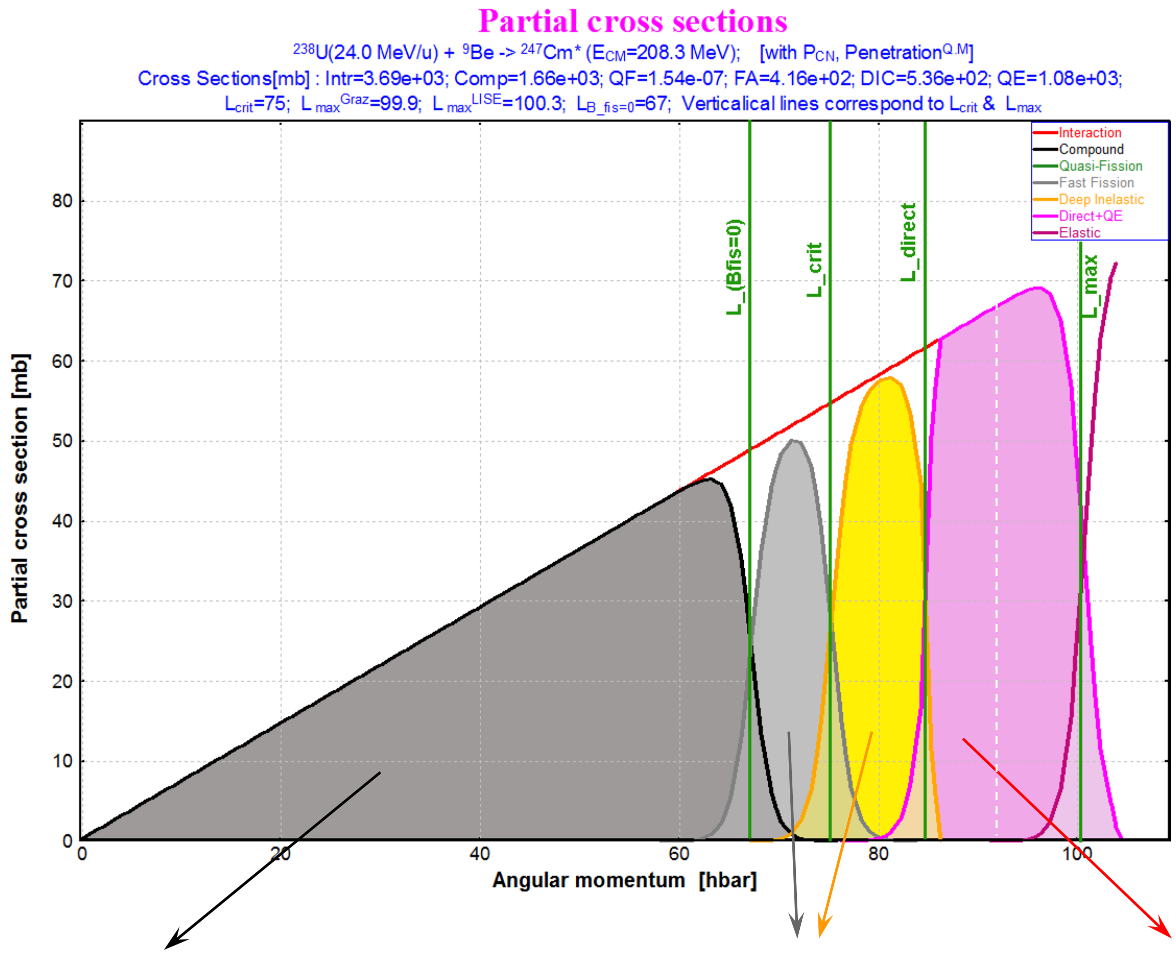
Momentum (hbar)	Be	C
L (Bfis=0)	67	63
L critical	75	87
L direct	79 (12)	101 (38)
L max (grazing)	90.5	118.9
L max (LISE)	91.0	119.5



Carbon target.. 50% split… Why?

This is due to difference of moments of inertia between C+U and Be+U just above where fission barrier go to zero

Fission channels for ^{238}U (20 MeV/u) + Be,C reactions



Compound fission ~100%
Fissile Z = 96
High Excitation Energy

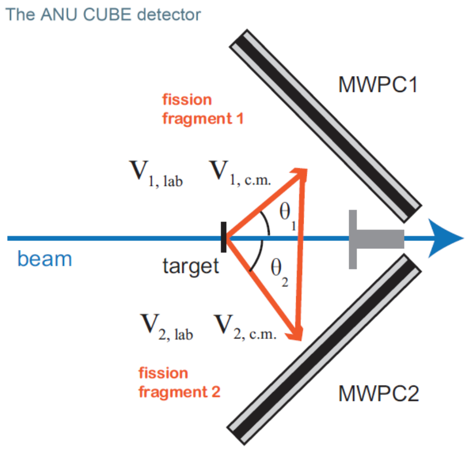
Sequential fission after DIC
Fissile Z < 92
High Excitation Energy

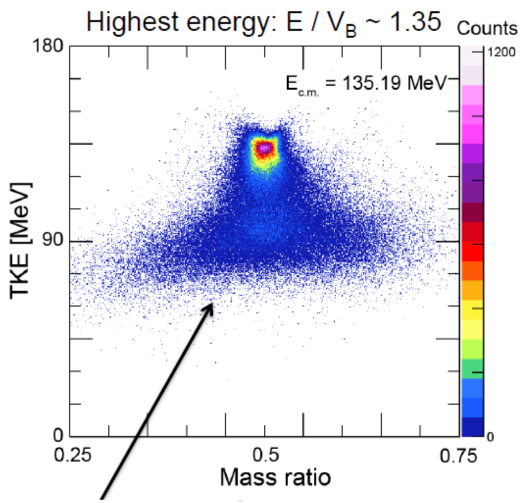
Partially go to fission
Fissile Z~92
Low Excitation Energy

Experimental study of barrier distributions for $^{58}Ni+^{60}Ni$

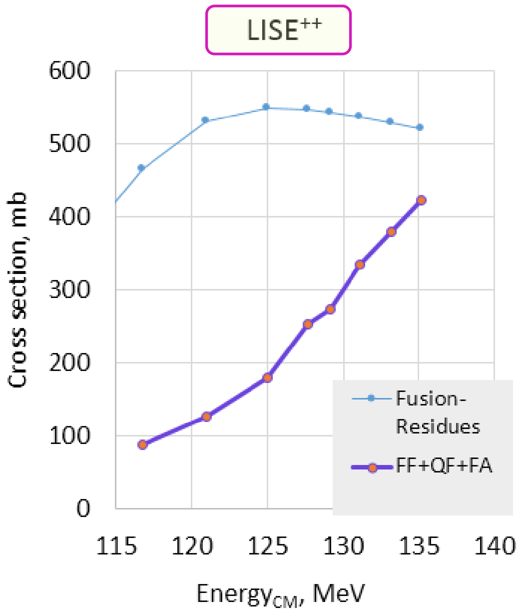
http://aip2014.org.au/cms/uploads/presentation/elizabeth_williams.pdf

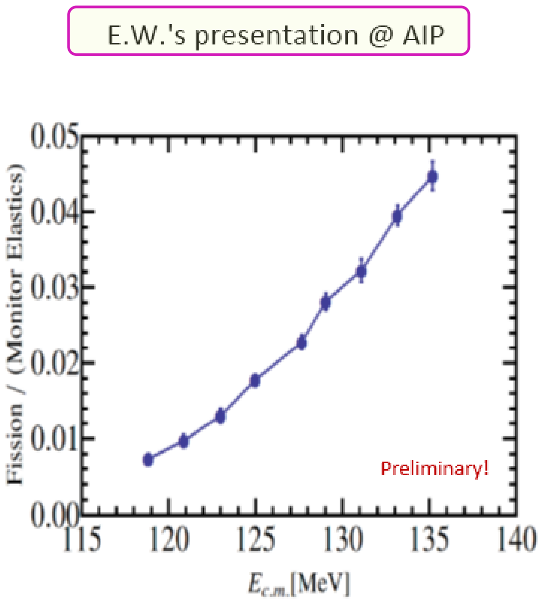
Dr. Elizabeth Williams
Department of Nuclear Physics
The Australian National University, Canberra, ACT, Australia
AIP Congress, Canberra, ACT
9 December 2014





Deep inelastic + Fusion-fission
- Fusion-fission onset at $E_{c.m.}$ ~ 125 MeV



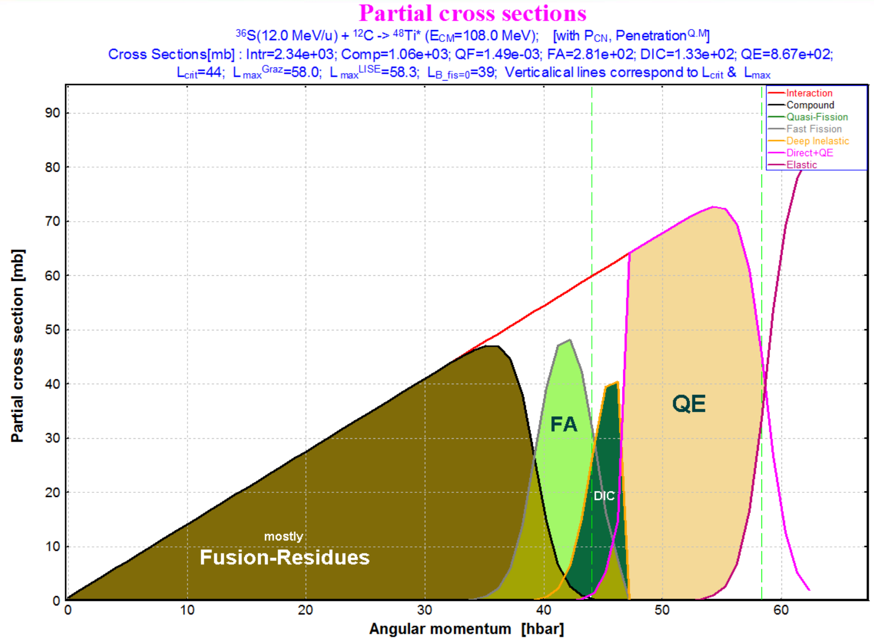


^{36}S (12 MeV/u) + ^{12}C, ^{238}U

MICHIGAN STATE UNIVERSITY

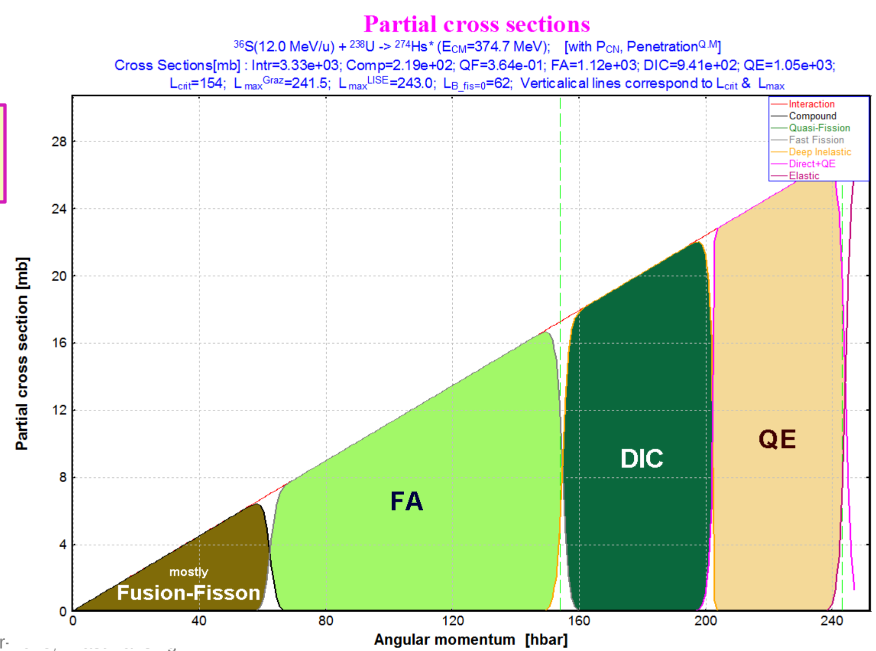
LISE++

^{12}C



Cross sections (mb)		
Partial (LISE++)		
Interaction	2.338e+03	
Compound	1.057e+03	
Quasi-Fission	1.494e-03	
Fast Fission	2.809e+02	
Deep Inelastic	1.334e+02	
Direct+QE	8.671e+02	
Compound 1st step de-excitation channels (LISE++)		
Fusion-Residue	8.678e+02	
Fusion-Fission	1.890e+02	
Fusion-Breackup	0.000e+00	
Bass cross section calculations		
Fusion cross section	1.044e+03	mb
Fusion barrier	14.49	MeV
Fusion radius	5.50	fm
Barrier position	8.80	fm
Grazing Angle (deg)		
Center-of-mass system	8.60	
Laboratory system	2.14	

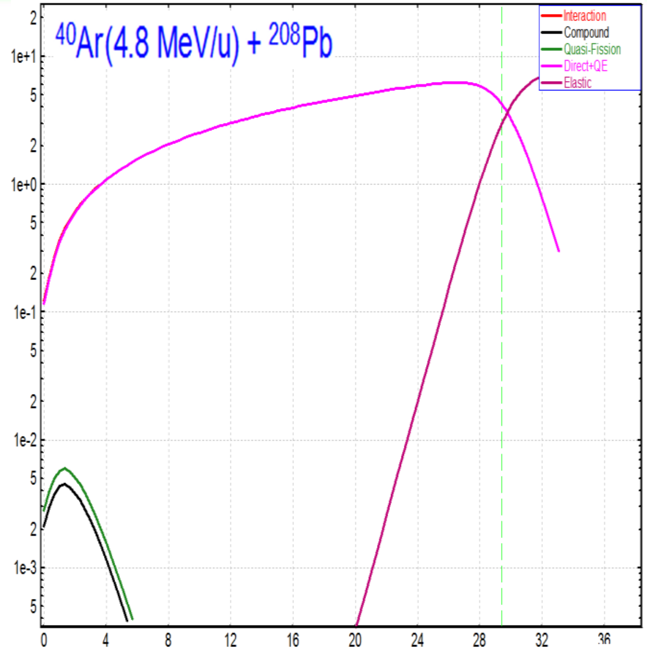
^{238}U

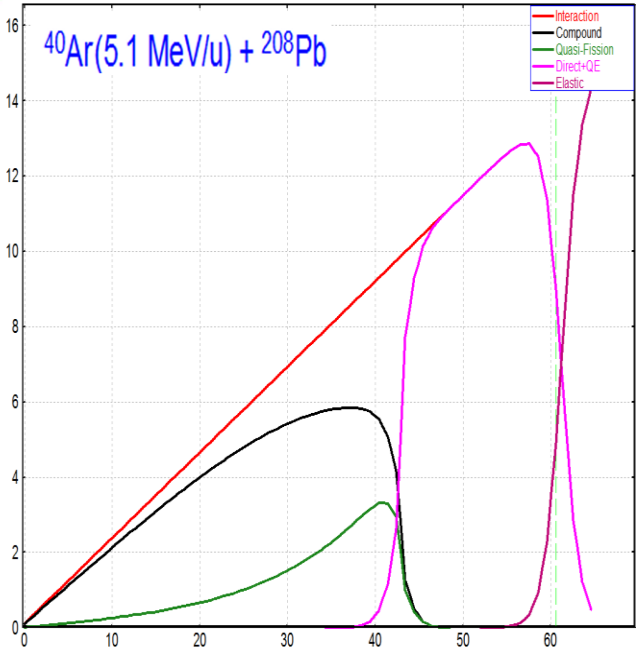


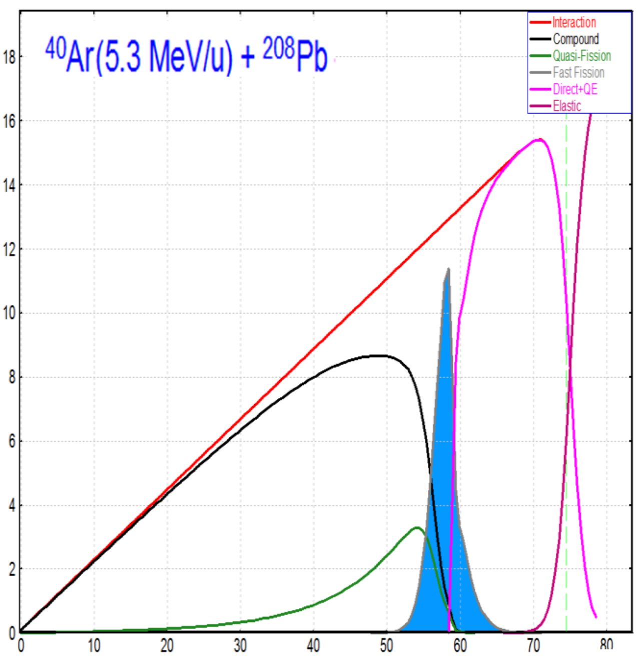
Cross sections (mb)		
Partial (LISE++)		
Interaction	3.328e+03	
Compound	2.191e+02	
Quasi-Fission	3.641e-01	
Fast Fission	1.121e+03	~2.04e3mb
Deep Inelastic	9.407e+02	
Direct+QE	1.047e+03	
Compound 1st step de-excitation channels (LISE++)		
Fusion-Residue	1.772e+01	
Fusion-Fission	2.014e+02	
Fusion-Breackup	0.000e+00	
Bass cross section calculations		
Fusion cross section	2.116e+03	mb
Fusion barrier	159.12	MeV
Fusion radius	10.40	fm
Barrier position	12.40	fm
Grazing Angle (deg)		
Center-of-mass system	30.97	
Laboratory system	26.99	

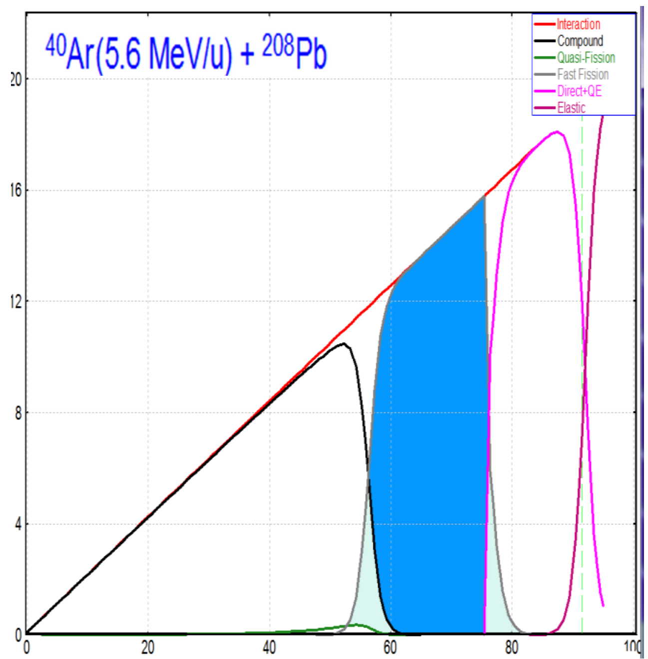
$^{40}Ar + ^{208}Pb$

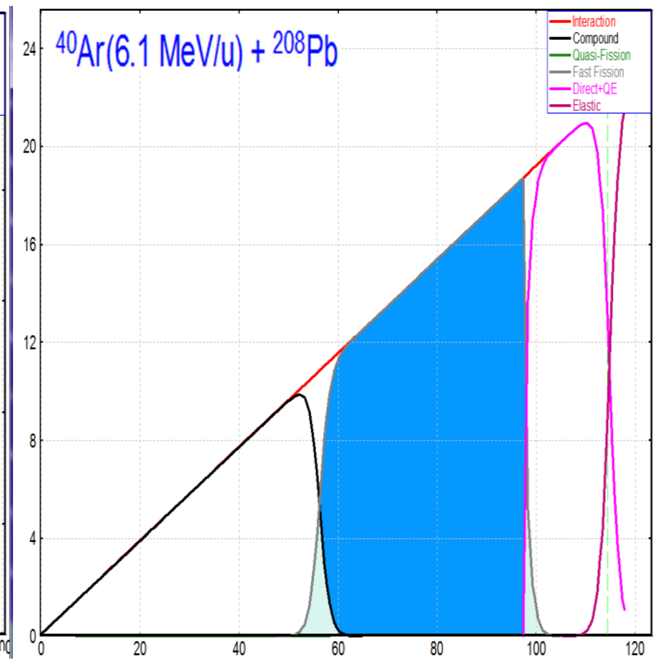
				cross sections				
Energy, Lab	Energy, CM	Bass barrier	ECM/ B -1	Compound	QF	FA	DI	QE
MeV/u	MeV	MeV	%	mb	mb	mb	mb	mb
4.8	160.8	161.9	**-0.7%**	1.5E-02	1.9E-02	0	0	1.1E+02
5.1	170.8	161.9	**5.5%**	1.6E+02	5.0E+01	0	0	2.2E+02
5.3	177.5	161.9	**9.7%**	3.1E+02	4.6E+01	4.4E+01	0	2.3E+02
5.6	187.6	161.9	**15.9%**	3.3E+02	5.0E+00	2.8E+02	0	2.7E+02
6.1	259.7	161.9	**60.4%**	3.1E+02	6.2E-01	6.2E+02	0	3.4E+02











Acknowledgements

I would like to acknowledge to

Profs. D.J.Morrissey, M.Thoennessenn,

Z.Kohley (NSCL/MSU),

Dr.G.Knyazheva (FLNR/Dubna),

for fruitful discussions.