Benchmark-weighted AA cross sections

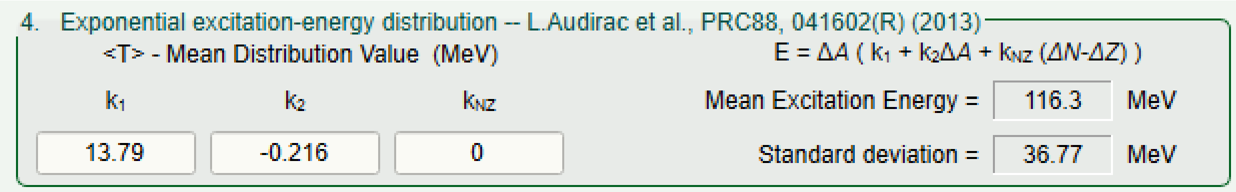
Created **09/01/25**
Updated **09/22/25 (slides 12-15)**

Experimental sets:

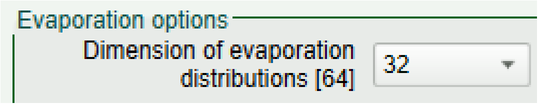
^{124}Xe(345AMeV) + Be : H.Suzuki et al., Prog. Theor. Exp. Phys. 2025 053D02
^{78}Kr(345AMeV) + Be : H.Suzuki et al., Prog. Theor. Exp. Phys. 2025 053D03

Which EE model is more approriate?

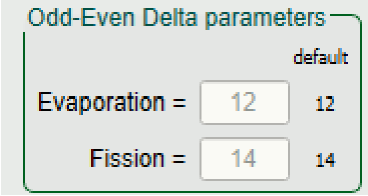
Excitation energy model "Tmean"
LISE v.17.15.27



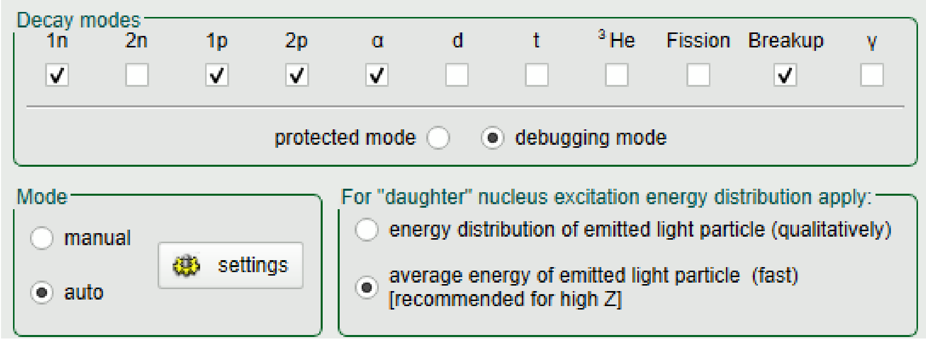
What is more suitable dimension value?



What are more suitable Odd/even values?



What decay channels should be used? What daughter energy distribution?



Deduction of Abrasion–Ablation (AA) Parameters and Weighted Cross Sections

Calibration on reference beams

The AA parameters (temperature coefficients, scaling factors, etc.) were determined separately for each of the eight nuclear mass models by fitting the measured production cross sections for the reactions ^{78}Kr + Be, ^{124}Xe + Be. Minimization employed the Levenberg–Marquardt algorithm with a combined χ^2/logarithmic objective to balance absolute and relative deviations.

Systematics for intermediate projectiles

For beams where no direct fit is possible (^{92}Mo), the AA parameters were interpolated/extrapolated from the excitation–energy systematics as a function of projectile mass and charge, using the trends established in the calibrated systems.

Mass-model ensemble

For each mass model, a complete set of AA parameters was thus available and used to calculate fragmentation cross sections for the beam of interest.

Model weighting

To combine the different mass models into a single prediction, relative statistical weights were assigned according to their performance in the reference systems:
$w_i \propto 1 / (\chi^2_{78Kr,i} \quad \chi^2_{124Xe,i})$ where χ^2 are the minimized objective values for mass model i.

Weighted averages

The final deduced cross sections were obtained as a weighted average across the eight mass models

Error estimation.

Cross-section uncertainties include both experimental errors and the spread among mass-model predictions, the latter quantified by the weighted variance.

Creation of CS user file to be used in LISE^{++}

^{78}Kr

Minimization value ↓ (Final)

Excitation energy parameters k_1, k_2, k_{NZ} (e0, e1, e2)

Limiting temperature to be used in break-up channel (Tlim)

Tunneling (dRauto)

date	method	mass	NP	s	2p	mw/2t	Final	wErr	e0	e1	e2	AA-factor	Tlim 50	Tlim 150		dRauto 0	dRauto 1	dRauto 2
29-Aug	Tmean	frdm2012	32	12	yes	2t	3.66	1.00	11.99	-2.5E-01	2.6E-01	1.00E+00	9.74E+00	6.00E+00		5.19E+00	-1.32E-01	4.56E-02
29-Aug	Tmean	tuyy	32	12	yes	2t	4.11	1.21	13.05	-8.9E-02	-3.4E-02	1.00E+00	8.79E+00	5.88E+00		5.18E+00	-1.35E-01	5.76E-02
1-Sep	Tmean	hfb27	32	12	yes	2t	5.13	1.57	12.94	-1.0E-01	-2.6E-03	1.00E+00				5.65E+00	-3.78E-02	3.50E-02
31-Aug	Tmean	ws4rbf	32	10	yes	2t	5.83	1.78	13.08	-1.1E-01	-1.7E-02					5.41E+00	-2.61E-01	7.47E-02
29-Aug	Tmean	ame2020	32	12	yes	2t	7.58	2.22	12.67	3.8E-02	-2.1E-01	1.00E+00	8.22E+00	6.31E+00		5.55E+00	3.65E-01	-9.51E-02
29-Aug	Tmean	ktyu	32	12	yes	2t	7.88	2.28	12.34	7.1E-03	-2.7E-01	1.00E+00	8.27E+00	6.03E+00		5.28E+00	3.67E-02	1.00E-01
29-Aug	Tmean	hfb22	32	12	yes	2t	8.67	2.45	13.29	-8.1E-02	-7.3E-02	1.00E+00	8.03E+00	5.94E+00		5.19E+00	-1.32E-01	4.61E-02
29-Aug	Tmean	unedf1	32	12	yes	2t	9.71	2.66	14.14	-3.5E-01	1.7E-01	1.00E+00	8.79E+00	5.88E+00		6.00E+00	-3.52E-02	9.80E-02
							meanW		*12.693*	*-0.142*	*0.048*		*9.045*	*5.983*		*5.337*	*-0.086*	*0.046*
							sigmaW		*0.217*	*0.039*	*0.064*		*0.283*	*0.051*		*0.084*	*0.052*	*0.016*

Line	Last	chi2 local	LogDif local	chi2 total	LogDif total
Z=30	18	0.1	2	0.2	10
Z=30	18	0.1	2	0.2	10
Z=30	21	0.1	2	0.1	5
Z=30	21	0.1	2	0.1	5

^{124}Xe

Minimization value (Final)

Excitation energy parameters k_1, k_2, k_{NZ}

Limiting temperature to be used in break-up channel

Tunneling

Beam	A	Z	date	method	mass	NP	s	2p	mw/2t	Final	wErr	e0	e1	e2	AA-factor	Tlim 50	Tlim 150		dRauto 0	dRauto 1	dRauto 2
124Xe	124	54	29-Aug	Tmean	hfb27	32	12	yes	2t	2.13	1.00	20.760	-2.61E-01	1.36E-01	1.00E+00	9.23E+00	6.89E+00		4.94E+00	-7.72E-02	4.12E-03
124Xe	124	54	29-Aug	Tmean	ws4rbf	32	12	yes	2t	2.87	1.32	20.148	-1.02E-01	5.39E-02	1.00E+00	8.71E+00	6.83E+00		5.20E+00	-1.95E-01	1.93E-02
124Xe	124	54	29-Aug	Tmean	unedf1	32	12	yes	2t	2.89	1.33	21.719	-1.31E-01	-6.61E-02	1.00E+00	8.71E+00	6.21E+00		5.20E+00	-1.71E-01	1.81E-02
124Xe	124	54	29-Aug	Tmean	ame2020	32	12	yes	2t	3.05	1.38	22.174	-9.83E-02	-5.75E-02	1.00E+00	8.27E+00	6.35E+00		4.84E+00	-4.68E-02	-3.57E-04
124Xe	124	54	29-Aug	Tmean	frdm2012	32	12	yes	2t	3.20	1.44	20.398	-2.07E-02	-2.05E-02	1.00E+00	8.00E+00	6.66E+00		4.77E+00	-1.20E-01	1.67E-02
124Xe	124	54	29-Aug	Tmean	hfb22	32	12	yes	2t	3.40	1.51	23.412	-2.44E-01	-2.71E-01	1.00E+00	1.01E+01	6.24E+00		4.55E+00	-1.30E-01	4.68E-02
124Xe	124	54	29-Aug	Tmean	ktyu	32	12	yes	2t	3.68	1.60	20.707	-3.13E-02	1.79E-01	1.24E+00	8.61E+00	6.81E+00		5.19E+00	2.30E-02	-2.19E-02
124Xe	124	54	29-Aug	Tmean	tuyy	32	12	yes	2t	4.30	1.78	21.222	1.25E-01	-2.58E-02	1.79E+00	8.03E+00	6.24E+00		5.18E+00	-1.64E-01	3.36E-02
										meanW		*21.238*	*-0.125*	*0.007*		*8.783*	*6.579*		*4.978*	*-0.109*	*0.013*
										sigmaW		*0.366*	*0.042*	*0.048*		*0.231*	*0.108*		*0.082*	*0.024*	*0.007*

Line	Last	chi2 local	LogDif local	chi2 total	LogDif total
Z=48	40	1	5	0.5	8

^{78}Kr & ^{124}Xe results product

	e1	e2	AA-factor	Tlim 50	Tlim 150		dRauto 0	dRauto 1	dRauto 2
meanW	*-0.134*	*0.022*		***8.888***	***6.090***		***5.152***	***-0.105***	***0.01812***
sigmaW	*0.009*	*0.019*		*0.128*	*0.229*		*0.179*	*0.009*	*0.012*

e2 (k_{ZN}) – can be ignored

RESULT -product								
method	mass	NP	s	2p	mw/2t	product	wErr	weight
Tmean	hfb27	32	12	yes	2t	10.93	1.00	1.000
Tmean	frdm2012	32	12	yes	2t	11.70	1.18	0.723
Tmean	ws4rbf	32	10	yes	2t	16.75	1.98	0.256
Tmean	tuyy	32	12	yes	2t	17.66	2.09	0.229
Tmean	ame2020	32	12	yes	2t	23.09	2.66	0.141
Tmean	unedf1	32	12	yes	2t	28.07	3.09	0.105
Tmean	ktyu	32	12	yes	2t	28.97	3.17	0.100
Tmean	hfb22	32	12	yes	2t	29.47	3.20	0.097

How to get Excitation energy parameter for ^{92}Mo

A practical form that matches your plot

Baseline (your V1):

$$\boxed{\varepsilon_0(A) = 13.5\left(\frac{A}{40}\right)^{1/3}\ \text{MeV per abraded}}$$

Optional, mild isospin tilt (captures the proton-rich vs neutron-rich trend seen in BeAGLE/INCL):

$$\boxed{\varepsilon(A,Z) = \varepsilon_0(A)\,[1 + c\,I],\quad I = \frac{N-Z}{A}}$$

- c is small (typ. $0-0.3$). Fit c once to your compiled BeAGLE/INCL + data; it will be close to zero for near-stability projectiles and grows slightly for more neutron-rich beams. (If you don't want any Z-dependence, set $c = 0$ and you're exactly on the purple V1 curve.)

Then

$$E^*(A,Z;\Delta A) = \varepsilon(A,Z)\,\Delta A.$$

Example: predict for a proton-rich $^{92}\mathrm{Mo}$ beam

$A = 92,\ Z = 42,\ N = 50,\ I = \frac{8}{92} = 0.087.$

- Without isospin term (your V1 only):

$$\varepsilon_0(92) = 13.5\left(\frac{92}{40}\right)^{1/3} \approx \mathbf{17.82\ MeV/abraded}.$$

- With a modest isospin correction (say $c = 0.2$):

$$\varepsilon(92,42) \approx 17.82 \times (1 + 0.2 \times 0.087) \approx \mathbf{18.13\ MeV/abraded}.$$

Notation

- $A^{1/3}$ baseline (your V1): $f(A) = \left(\frac{A}{40}\right)^{1/3}$.
- Asymmetry $I = (N - Z)/A$.
  - $I_{78} = \frac{6}{78} = 0.076923$, $I_{124} = \frac{16}{124} = 0.129032$, $I_{92} = \frac{8}{92} = 0.086957$.
- Measured (from your table, per mass model):
  - $y_{78} \equiv e0(^{78}\mathrm{Kr})$ (≈12–13)
  - $y_{124} \equiv e0(^{124}\mathrm{Xe})$ (≈21–22)

Model (what I suggested)

$$\varepsilon(A,Z) = k\ f(A)\,[1 + c\,I]$$

- k is the overall scale (your "e0 at $A = 40$" in effect),
- c is a small dimensionless **isospin tilt** (your "e1").

Solve k, c for each mass model (from the two beams)

Let $a = f(78),\ b = f(124)$ (numerically $a = 1.249333,\ b = 1.458100$).

1. Compute

$$r = \frac{y_{124}}{y_{78}} \cdot \frac{a}{b}$$

2. Then

$$\boxed{c = \frac{1 - r}{r\,I_{78} - I_{124}}},\qquad \boxed{k = \frac{y_{78}}{a\,(1 + c\,I_{78})}}$$

Predict for 92Mo (A=92, Z=42)

Let $f_{92} = f(92) = 1.333454$ and $I_{92} = 0.086957$:

$$\boxed{e0(^{92}\mathrm{Mo}) = \varepsilon_{92} = k\ f_{92}\,(1 + c\,I_{92})}$$

How to get Excitation energy parameter for ^{92}Mo

78Kr_124Xe.xlsx

Sum of e0	Column Labels					e0	
	78Kr	124Xe				92Mo	144Sm
	78	124				92	144
Row Labels	36	54	R	C	k	42	62
ame2020	12.668	22.174	0.67	36.57	2.66	14.673	24.776
frdm2012	11.994	20.398	0.69	26.99	3.12	13.788	22.713
hfb22	13.288	23.412	0.66	39.48	2.63	15.417	26.180
hfb27	12.941	20.76	0.73	16.07	4.63	14.659	22.946
ktyu	12.336	20.707	0.70	23.82	3.49	14.134	23.020
tuyy	13.051	21.222	0.72	17.99	4.38	14.833	23.498
unedf1	14.136	21.719	0.76	11.40	6.03	15.846	23.867
ws4rbf	13.083	20.148	0.76	11.61	5.53	14.674	22.148
Isospin factor	0.0769	0.1290				0.0870	0.1389
fA	1.24933	1.45810				1.32001	1.53262

Sum of e1	Column Labels				e1	
	78Kr	124Xe			92Mo	144Sm
	78	124			92	144
Row Labels	36	54	K	B	42	62
ame2020	0.038	-0.098	-3.0E-03	2.7E-01	-3.40E-03	-1.58E-01
frdm2012	-0.253	-0.021	2.6E-03	-4.6E-01	-2.16E-01	-7.88E-02
hfb22	-0.081	-0.244	-1.3E-03	2.2E-02	-9.96E-02	-1.68E-01
hfb27	-0.100	-0.261	-1.3E-03	1.3E-03	-1.18E-01	-1.86E-01
ktyu	0.007	-0.031	-3.1E-04	3.1E-02	2.72E-03	-1.34E-02
tuyy	-0.089	0.125	1.7E-03	-2.2E-01	-6.46E-02	2.49E-02
unedf1	-0.348	-0.131	1.7E-03	-4.8E-01	-3.23E-01	-2.32E-01
ws4rbf	-0.108	-0.102	5.0E-05	-1.1E-01	-1.08E-01	-1.05E-01

Average-Weighted result

92Mo_summary2.xlsm

Z=36, N=32

Working in log-space

Regular average | Weighted average | Asymmetric Weighted average | Final result in linear space

									count	average	stDev	averageW	sigmaW	asym	asym er-min	asym err-plus	CS	dCS-	dCS+
wErr	1.000	1.176	1.978	2.090	2.660	3.093	3.165	3.205											
weight	1.000	0.723	0.256	0.229	0.141	0.105	0.100	0.097											
err	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1											
Sum of logValue	Column Labels																		
Row Labels	ame2020	frdm2012	hfb22	hfb27	ktyu	tuyy	unedf1	ws4rbf											
36032	-11.2124	-9.4153	-11.1029	-10.9287	-11.4284	-11.1956	-8.7143	-12.0695	8	-10.7584	1.0425	-10.6354	0.3424	-10.6362	0.6767	0.9831	2.31E-11	1.82E-11	1.99E-10
36033	-9.4572	-7.8130	-8.9366	-8.0534	-9.2344	-7.6570	-6.9229	-9.3183	8	-8.4241	0.8760	-8.6538	0.3074	-8.8552	0.6605	0.8806	1.40E-09	1.09E-09	9.21E-09
36034	-7.9198	-6.2975	-7.1237	-7.7824	-7.2311	-7.6533	-5.6909	-7.4676	8	-7.1458	0.7254	-7.2409	0.2784	-7.3577	0.6225	0.8565	4.39E-08	3.34E-08	2.72E-07
36035	-5.4290	-4.5121	-5.2342	-5.1552	-5.3199	-5.5366	-4.1907	-5.2961	8	-5.0842	0.4440	-5.0834	0.1601	-5.1208	0.3437	0.4905	7.57E-06	4.14E-06	1.59E-05
36036	-3.2756	-2.9083	-2.9701	-4.2258	-3.4554	-3.8397	-3.2157	-3.8470	8	-3.4672	0.4356	-3.2786	0.1465	-3.1883	0.2984	0.2311	6.48E-04	3.22E-04	4.55E-04
36037	-1.9324	-1.6059	-1.7057	-2.1497	-1.9316	-2.3697	-1.7792	-1.7183	8	-1.8991	0.2390	-1.8438	0.0760	-1.8302	0.1548	0.1836	1.48E-02	4.43E-03	7.78E-03
36038	-0.9234	-0.6479	-0.6035	-1.2739	-1.0352	-1.3781	-0.9113	-1.0194	8	-0.9741	0.2523	-0.8746	0.0828	-0.8414	0.1607	0.1697	1.44E-01	4.46E-02	6.89E-02
36039	0.0096	0.2285	0.1347	0.1217	0.0753	-0.1141	0.1608	0.1818	8	0.0998	0.1020	0.1017	0.0377	0.0874	0.0946	0.1182	1.22E+00	2.39E-01	3.82E-01
36040	0.5642	0.6765	0.8021	1.2261	0.3143	0.7476	0.7236	1.2743	8	0.7911	0.3002	0.7010	0.0863	0.6318	0.0950	0.1776	4.28E+00	8.42E-01	2.16E+00
36041	1.1360	1.0849	0.8176	1.2020	1.1947	1.1516	1.2013	1.3382	8	1.1408	0.1401	1.1107	0.0415	1.1122	0.0918	0.0571	1.29E+01	2.47E+00	1.82E+00
36042	1.6154	1.2313	1.4885	1.3016	1.1926	1.2271	1.2768	1.2656	8	1.3249	0.1385	1.4079	0.0671	1.4706	0.2047	0.1615	2.96E+01	1.11E+01	1.33E+01
36043	1.0930	1.2330	1.0521	1.1285	1.1226	1.0780	1.1263	1.1362	8	1.1212	0.0503	1.1342	0.0242	1.1365	0.0640	0.0837	1.37E+01	1.88E+00	2.91E+00
36044	0.9371	1.1639	1.0838	1.0398	0.8540	0.8449	0.7262	0.8870	8	0.9421	0.1350	1.0042	0.0463	1.0130	0.1008	0.1256	1.03E+01	2.14E+00	3.45E+00
36045	0.3532	0.4538	0.3216	0.4522	0.3639	0.2921	0.2330	0.3395	8	0.3512	0.0701	0.3793	0.0231	0.3851	0.0564	0.0667	2.43E+00	2.96E-01	4.03E-01
36046	-0.0212	-0.0336	-0.2092	0.6936	-0.1242	-0.0965	-0.2261	-0.0247	8	-0.0052	0.2745	0.0027	0.0840	-0.0131	0.0740	0.1795	9.70E-01	1.52E-01	4.97E-01
36047	-0.7515	-0.8066	-0.9230	-0.7378	-0.8641	-0.8367	-0.9819	-0.6835	8	-0.8231	0.0930	-0.7974	0.0259	-0.7776	0.0642	0.0475	1.67E-01	2.29E-02	1.93E-02
36048	-1.2751	-1.3499	-1.5408	-1.4438	-1.5582	-1.4943	-1.6816	-1.4259	8	-1.4712	0.1189	-1.3803	0.0430	-1.3217	0.0986	0.0618	4.77E-02	9.68E-03	7.29E-03
36049	-2.4407	-2.5264	-2.7399	-2.4614	-2.4512	-2.4147	-2.6140	-2.3917	8	-2.5050	0.1104	-2.4990	0.0349	-2.4801	0.0895	0.0572	3.31E-03	6.16E-04	4.66E-04
36050	-3.8823	-3.6445	-4.2062	-3.6739	-3.7614	-3.7710	-4.0167	-3.7533	8	-3.8387	0.1772	-3.8201	0.0632	-3.8127	0.1394	0.1417	1.54E-04	4.23E-05	5.94E-05
37031	-13.5459	-12.2580	-15.0838	-13.5787	-13.2272	-14.0641	-11.6836	-6.2325	8	-12.4592	2.5471	-13.0104	0.5982	-13.1558	0.7004	1.0308	6.99E-14	5.59E-14	6.80E-13
37032	-12.8150	-11.4233	-13.5927	-12.9762	-13.4145	-13.2025	-10.6594	-13.8294	8	-12.7391	1.0433	-12.5276	0.3261	-12.4223	0.6016	0.8094	3.78E-13	2.84E-13	2.06E-12
37033	-10.9077	-9.5353	-11.2279	-9.8677	-10.8634	-10.5377	-8.7854	-10.9498	8	-10.3344	0.7971	-10.3791	0.2707	-10.4483	0.5521	0.7551	3.56E-11	2.56E-11	1.67E-10
37034	-9.9019	-8.5199	-10.0951	-9.7625	-9.7424	-9.5255	-7.5585	-10.0172	8	-9.3904	0.8326	-9.4241	0.2705	-9.4602	0.5236	0.7686	3.47E-10	2.43E-10	1.69E-09
37035	-7.6965	-6.6955	-8.0847	-6.0140	-7.5266	-7.5076	-5.8801	-6.1475	8	-6.9441	0.8076	-7.1737	0.2603	-7.3272	0.4563	0.6287	4.71E-08	3.06E-08	1.53E-07
37036	-5.8866	-5.0756	-6.1022	-5.5173	-5.8085	-6.0967	-4.7795	-5.7381	8	-5.6256	0.4463	-5.6113	0.1553	-5.6258	0.3210	0.4446	2.37E-06	1.24E-06	4.22E-06
37037	-2.9210	-3.1952	-4.2654	-3.0457	-3.1888	-3.4280	-3.2336	-2.4789	8	-3.2196	0.4752	-3.1660	0.1516	-3.0674	0.3531	0.1747	8.56E-04	4.76E-04	4.24E-04
37038	-2.4209	-1.8875	-2.4895	-2.6423	-2.1789	-2.7114	-2.1648	-2.1235	8	-2.3273	0.2658	-2.2791	0.1031	-2.2613	0.2168	0.2969	5.48E-03	2.15E-03	5.38E-03
37039	-0.7757	-0.6092	-1.1769	-0.9158	-0.6320	-0.7595	-0.8949	-0.4935	8	-0.7822	0.2008	-0.7669	0.0647	-0.7414	0.1395	0.1141	1.81E-01	4.98E-02	5.45E-02
37040	-0.1902	-0.0346	-0.0107	-0.2372	-0.0746	-0.1858	-0.1687	-0.0032	8	-0.1131	0.0865	-0.1205	0.0318	-0.1341	0.0754	0.0939	7.34E-01	1.17E-01	1.77E-01
37041	0.5837	0.7689	0.1731	0.3641	0.6967	0.5895	0.7575	0.6291	8	0.5703	0.1919	0.5901	0.0672	0.6209	0.1500	0.1231	4.18E+00	1.22E+00	1.37E+00
37042	0.9316	1.0878	1.7447	0.9299	0.9537	0.9509	1.0129	0.9848	8	1.0745	0.2580	1.0594	0.0882	1.0124	0.0977	0.2150	1.03E+01	2.07E+00	6.59E+00
37043	1.2529	1.3989	1.0674	1.1877	1.3001	1.2703	1.3311	1.3029	8	1.2639	0.0937	1.2772	0.0366	1.2902	0.0860	0.0926	1.95E+01	3.51E+00	4.64E+00
37044	1.2035	1.3993	1.2407	1.3228	1.2700	1.2412	1.2446	1.1308	8	1.2566	0.0743	1.2747	0.0323	1.2700	0.0824	0.1084	1.86E+01	3.22E+00	5.28E+00
37045	0.9788	1.2127	0.9751	0.9788	1.1018	0.9972	1.0681	1.1497	8	1.0578	0.0848	1.0592	0.0390	1.0540	0.0939	0.1303	1.13E+01	2.20E+00	3.96E+00
37046	0.6985	0.7680	0.7078	0.7551	0.7475	0.6844	0.5678	0.6826	8	0.7015	0.0592	0.7198	0.0166	0.7218	0.0471	0.0516	5.27E+00	5.41E-01	6.64E-01
37047	0.2188	0.2635	-0.0055	0.0536	0.2284	0.1242	0.1162	0.2637	8	0.1579	0.0945	0.1897	0.0332	0.2180	0.0838	0.0506	1.65E+00	2.90E-01	2.04E-01

LISE^{++} package v.17.16.1

Result of this analysis

Experimental CS used in this analysis

:\CrossSections\PublishedData*.*

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92Mo_C_BAA	cs	27,785	09/22/2025 17:27
124Xe_Be_345AMeV	cs	4,259	06/01/2025 01:30
136Xe_Be_1AGeV	cs	3,180	05/30/2025 23:24
78Kr_Be_345AMeV	cs	21,489	05/30/2025 18:25
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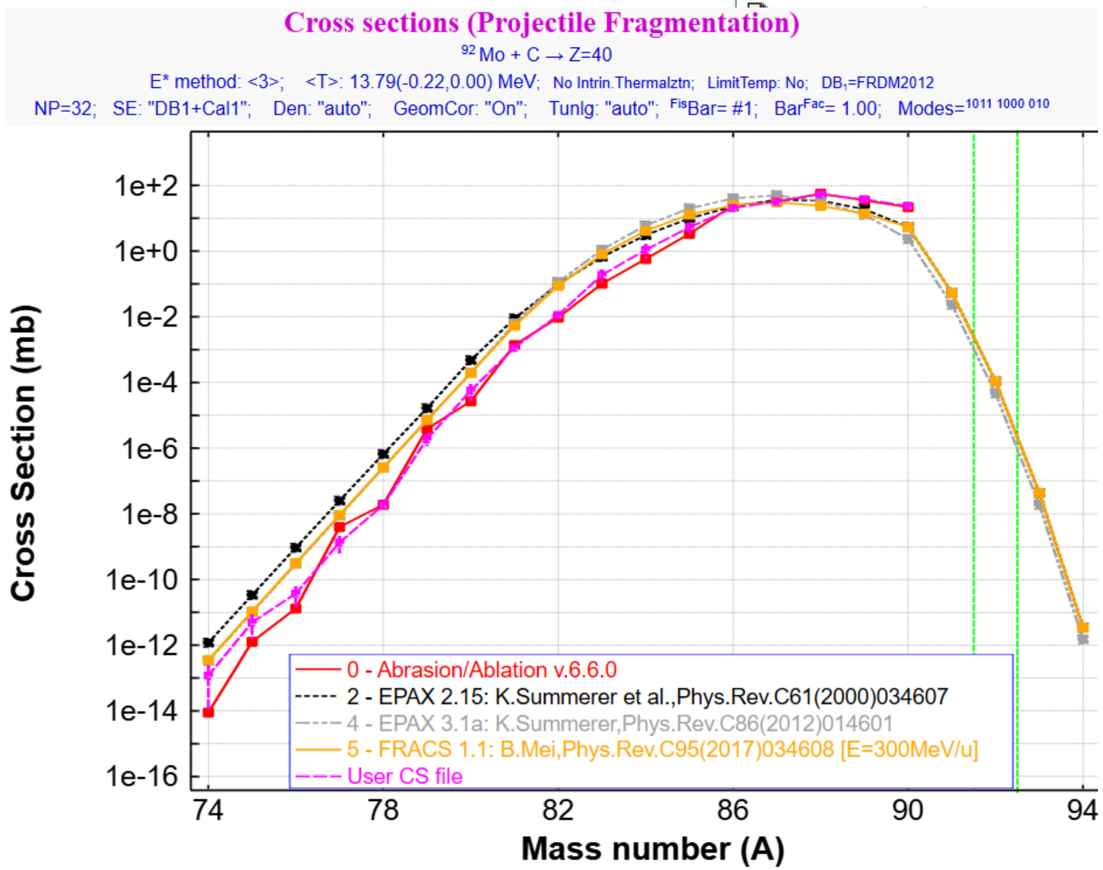
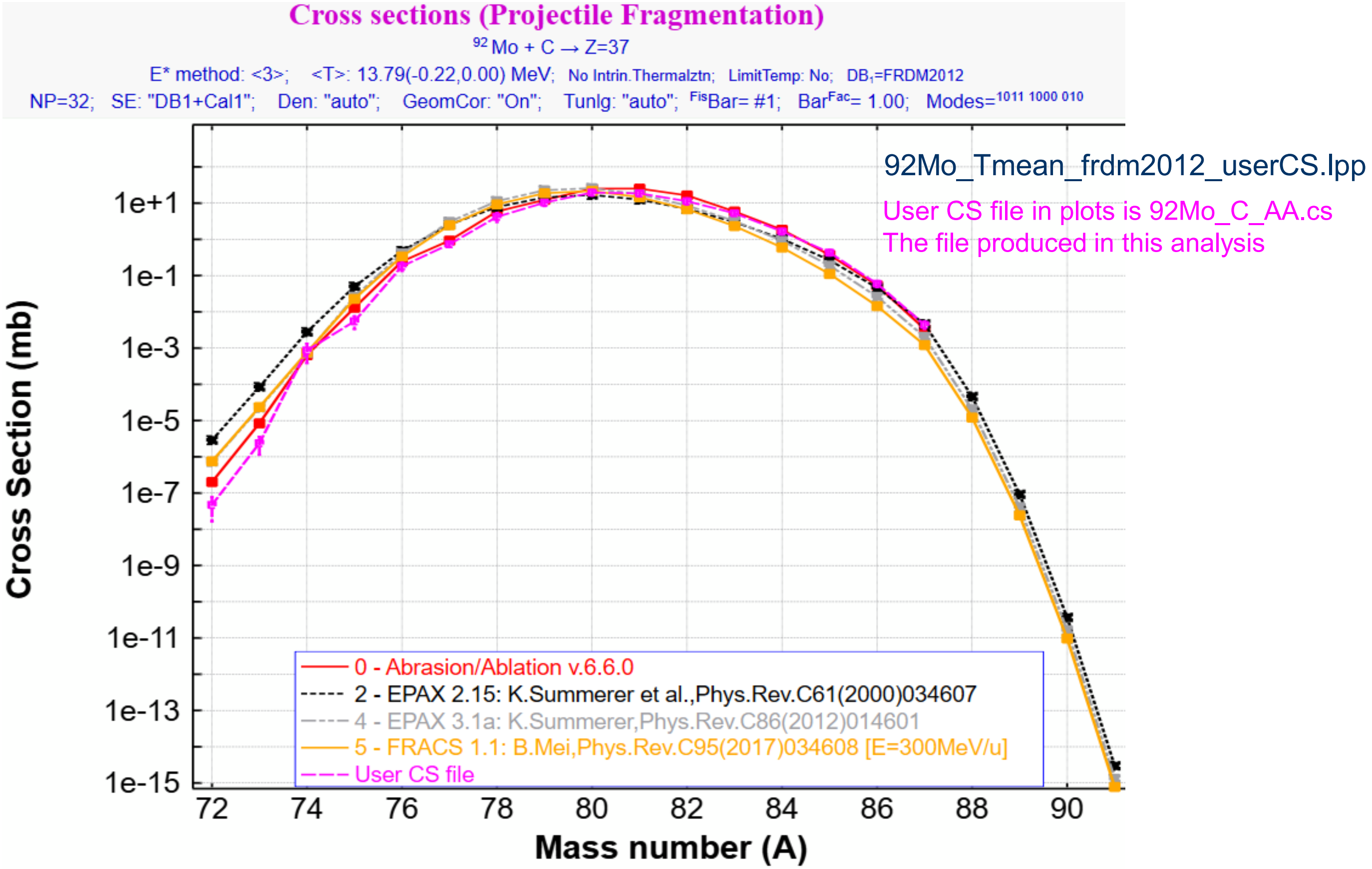
82Se_Be_140AMeV	cs	8,994	12/26/2013 12:44





ChatGPT comments regarding to this analysis

Title

“Weighted AA Ensemble Cross Sections”
“Model-averaged AA Predictions”
“Model-averaged AA cross sections”
“Benchmark-weighted AA cross sections”

Neutral/statistical
Weighted AA average cross sections
Model-averaged AA cross sections
Weighted model AA cross sections
AA cross sections from weighted model averaging (WMA)

Physics-oriented
Benchmark-calibrated AA cross sections
Reference-weighted AA cross sections
Data-guided AA average cross sections
AA cross sections with mass-model weighting

Slightly more formal
Weighted synthesis of AA cross sections
Aggregated AA predictions
Composite AA cross sections
Weighted AA estimate

1. Common practice vs new

•**Common practice pieces**:

- Calibrating AA parameters on one or two reference beams is standard.
- Comparing several mass models (FRDM, HFB, KTUY, WS4, …) is very common.
- Assigning weights based on χ^2 performance has been done in some mass-model systematics papers, usually under the name “weighted model average” or “Bayesian model averaging.”

•**Less common / novel piece**:

- In fragmentation cross-section systematics, most groups either (i) show a *band/spread* of different mass models, or (ii) pick one “best” mass model.
- Using *quantitative weights from multiple reference beams* to produce a single averaged AA prediction with propagated uncertainties is much less common.
- To my knowledge, this kind of weighted AA cross-section ensemble has **not yet appeared systematically in PRC/EPJA fragmentation papers.**

So you are not reinventing statistics, but you are **bringing an established idea (model averaging) into nuclear fragmentation systematics**, which makes it publishable.

2. Is it publishable?

Yes — absolutely:

- Referees will appreciate that you **do not cherry-pick one mass model**, but instead combine them transparently, with weights tied to real data.
- It gives **more robust extrapolations** to beams where no data exist.
- By publishing, you set a precedent: future groups can compare to your “ensemble AA” rather than debating mass-model choice.

The key will be to **emphasize**:

- The method is *data-driven* (fits to 78Kr and 124Xe).
- It *quantifies model uncertainty* via weighted variance.
- It produces a *single recommended cross section with error bar*.

That is valuable for experimental planning (where people need one number + error, not eight).

General Bayesian model averaging (BMA)

Bayesian model averaging (BMA) is exactly the "family resemblance" of what you already do with χ^2–based weights. Let me unpack it carefully for your AA cross-section context.

Bayesian Model Averaging (BMA) for Abrasion–Ablation Cross Sections

1. General Bayesian model averaging (BMA)

Suppose you have several models M_i (here: eight mass models + AA fits). In Bayesian statistics, each model has a posterior probability given the data D:

$$P(M_i|D) \propto P(D|M_i)\, P(M_i)$$

where
- $P(D|M_i)$ is the marginal likelihood (how well model M_i explains the data),
- $P(M_i)$ is the prior probability of model i.

The final prediction for an observable y is the model-averaged posterior predictive distribution:

$$P(y|D) = \Sigma_i\, P(y|M_i,D)\, P(M_i|D).$$

2. Connection to current approach

Your weights
$w_i \propto 1 / (\chi_{78Kr,i}{}^2 \times \chi_{124Xe,i}{}^2)$

are a frequentist proxy for $P(M_i|D)$. In full Bayesian form, you would instead compute $P(D|M_i) \propto \exp(-\chi_i{}^2/2)$, assuming Gaussian residuals. Then
$w_i = \exp(-\chi_i{}^2/2) / \Sigma_j \exp(-\chi_j{}^2/2)$.

3. Bayesian version for AA analysis

1. Model prior: assume equal priors for all eight mass models.
2. Likelihood: for each model, compute
   $L_i = \exp(-\chi^2{}_{78Kr,i}/2) \times \exp(-\chi^2{}_{124Xe,i}/2)$.
3. Posterior weight: $w_i = L_i / \Sigma_j L_j$.
4. Model-averaged prediction:
   $\langle\sigma\rangle = \Sigma_i\, w_i\, \sigma_i$,
   with uncertainty from the posterior variance.

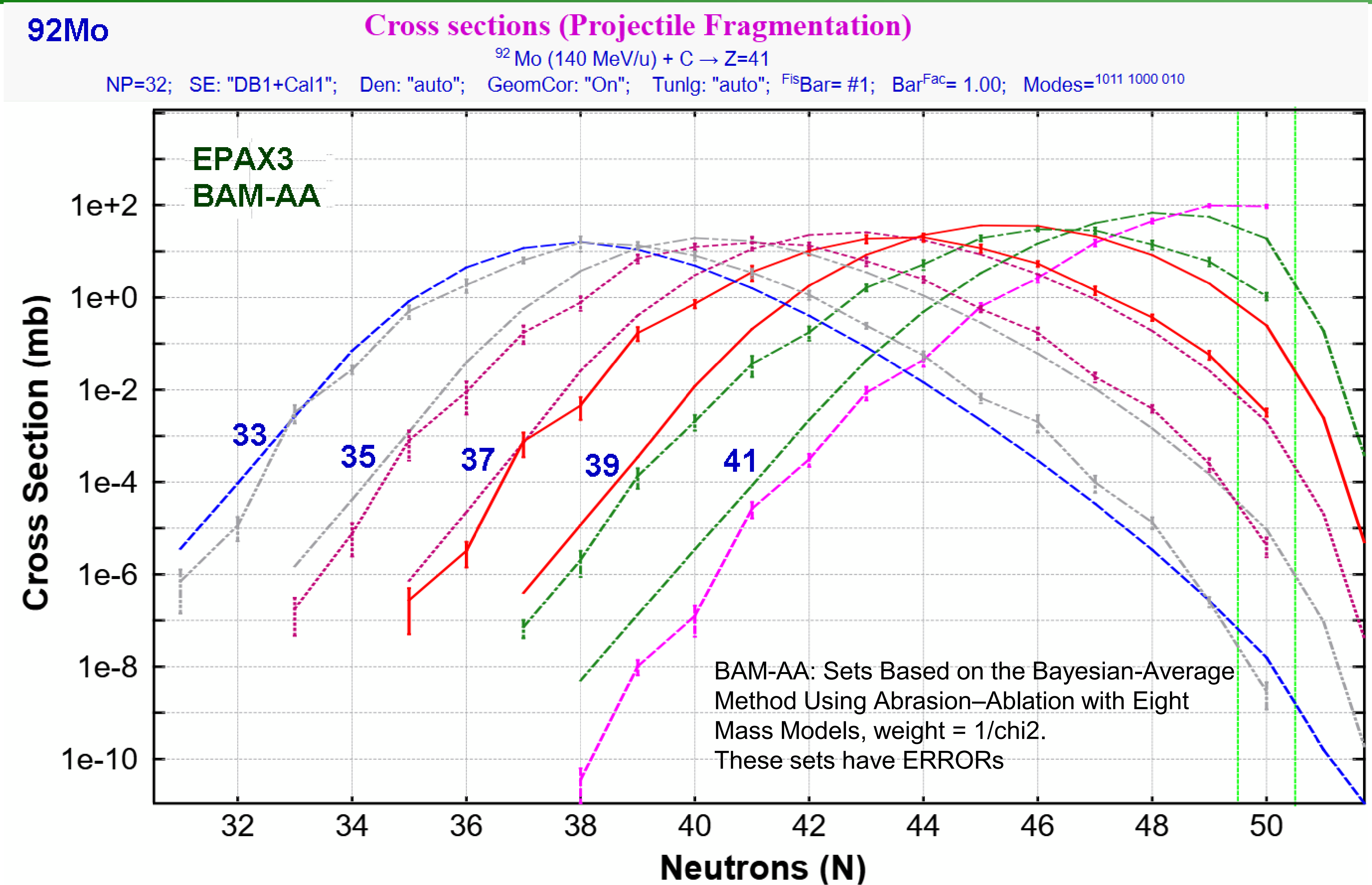
4. Advantages vs χ^2-based weights

- Theoretical grounding: weights correspond to probabilities.
- Automatic scaling: exponential likelihood prevents overweighting mediocre fits.
- Error bars: BMA naturally propagates both within-model error and between-model spread.

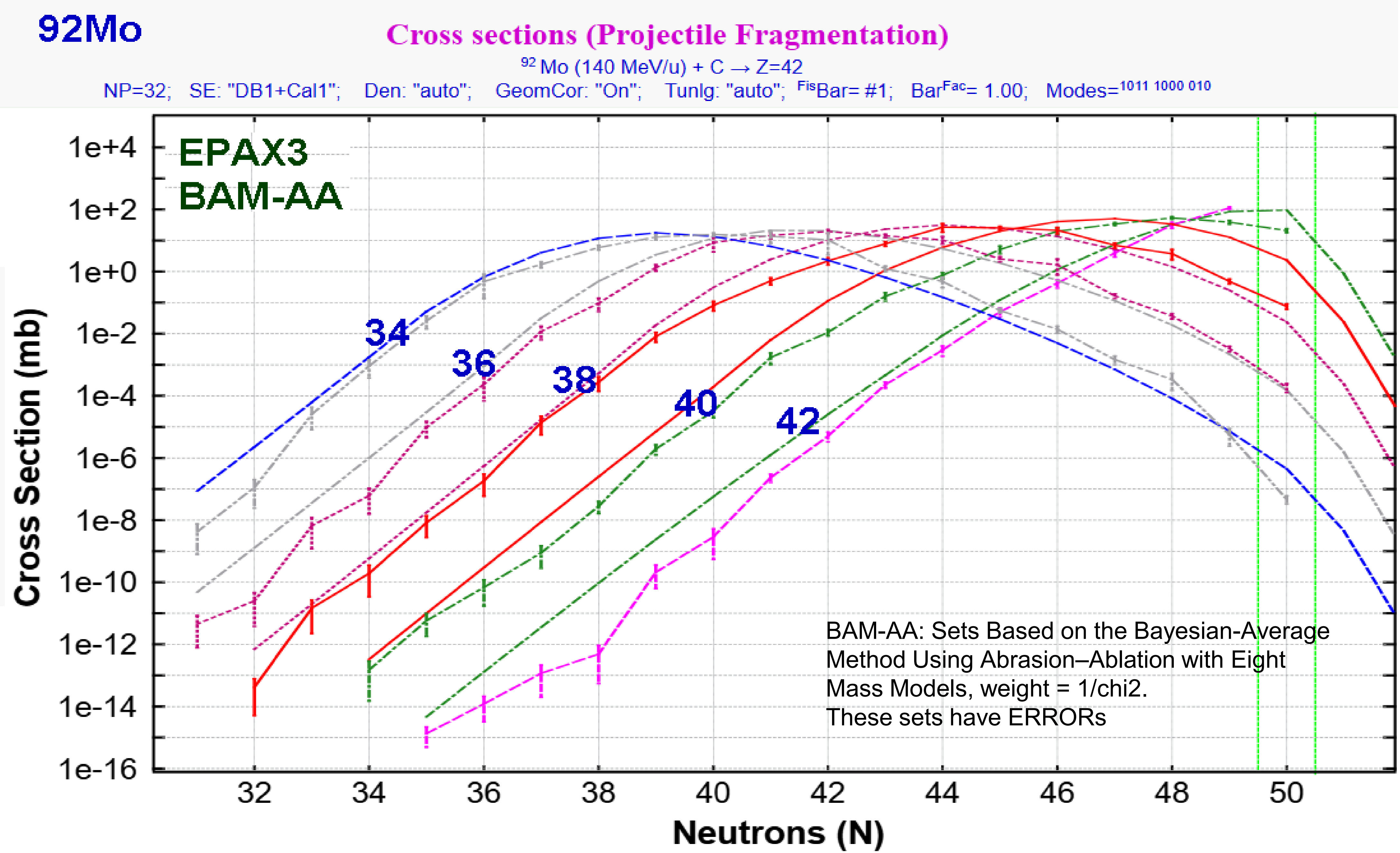
5. Suggested manuscript wording

We employed a weighted model averaging procedure, akin to Bayesian model averaging, to combine AA predictions based on eight nuclear mass models. In this framework, the weight of each mass model is proportional to the likelihood of reproducing the benchmark cross sections (^{78}Kr and ^{124}Xe), effectively assigning posterior probabilities to each model. The final deduced cross sections thus represent a model-averaged prediction with uncertainties reflecting both experimental input and inter-model spread.

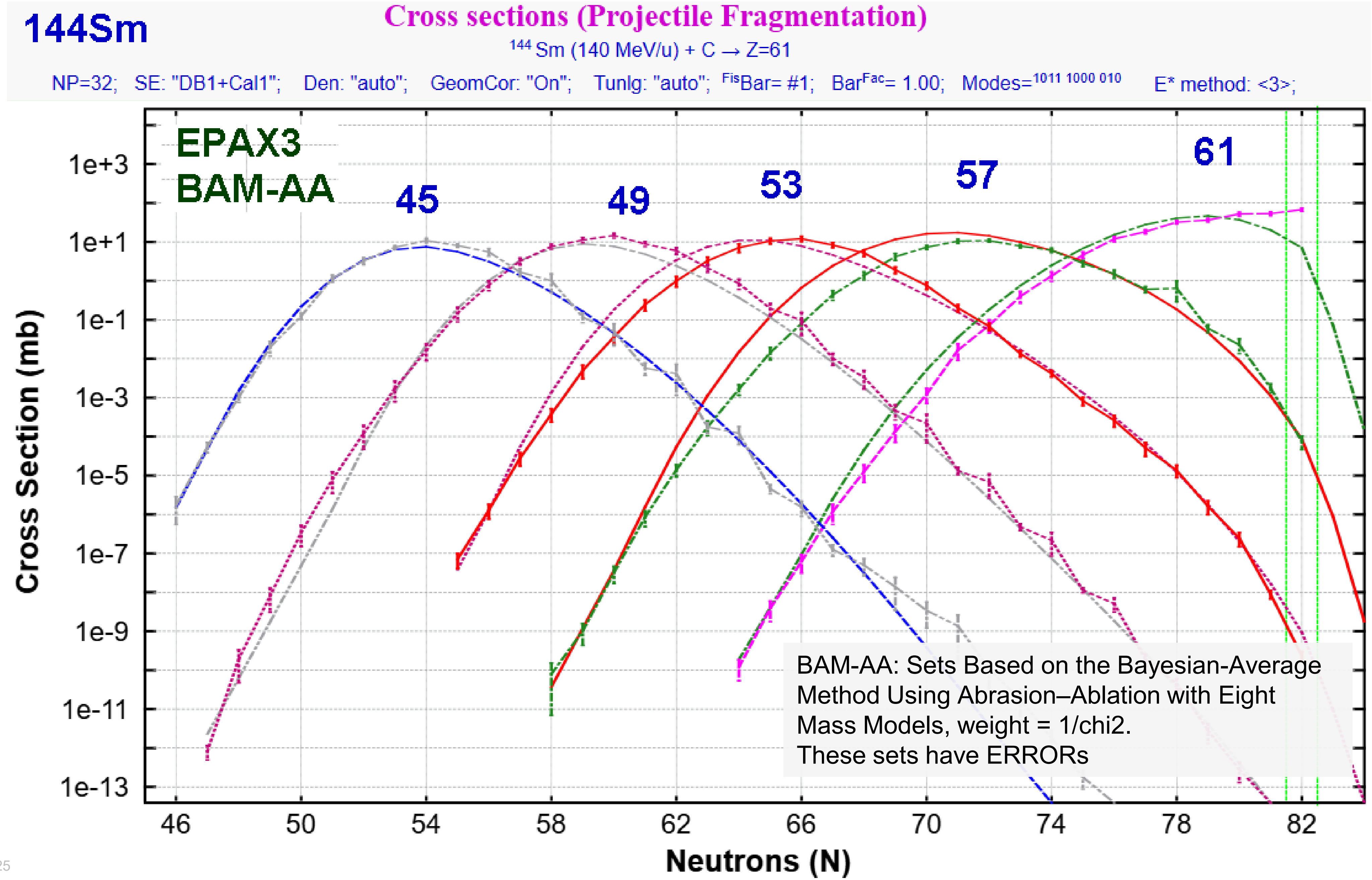
General Bayesian model averaging (BMA): ^{92}Mo → Z-odd



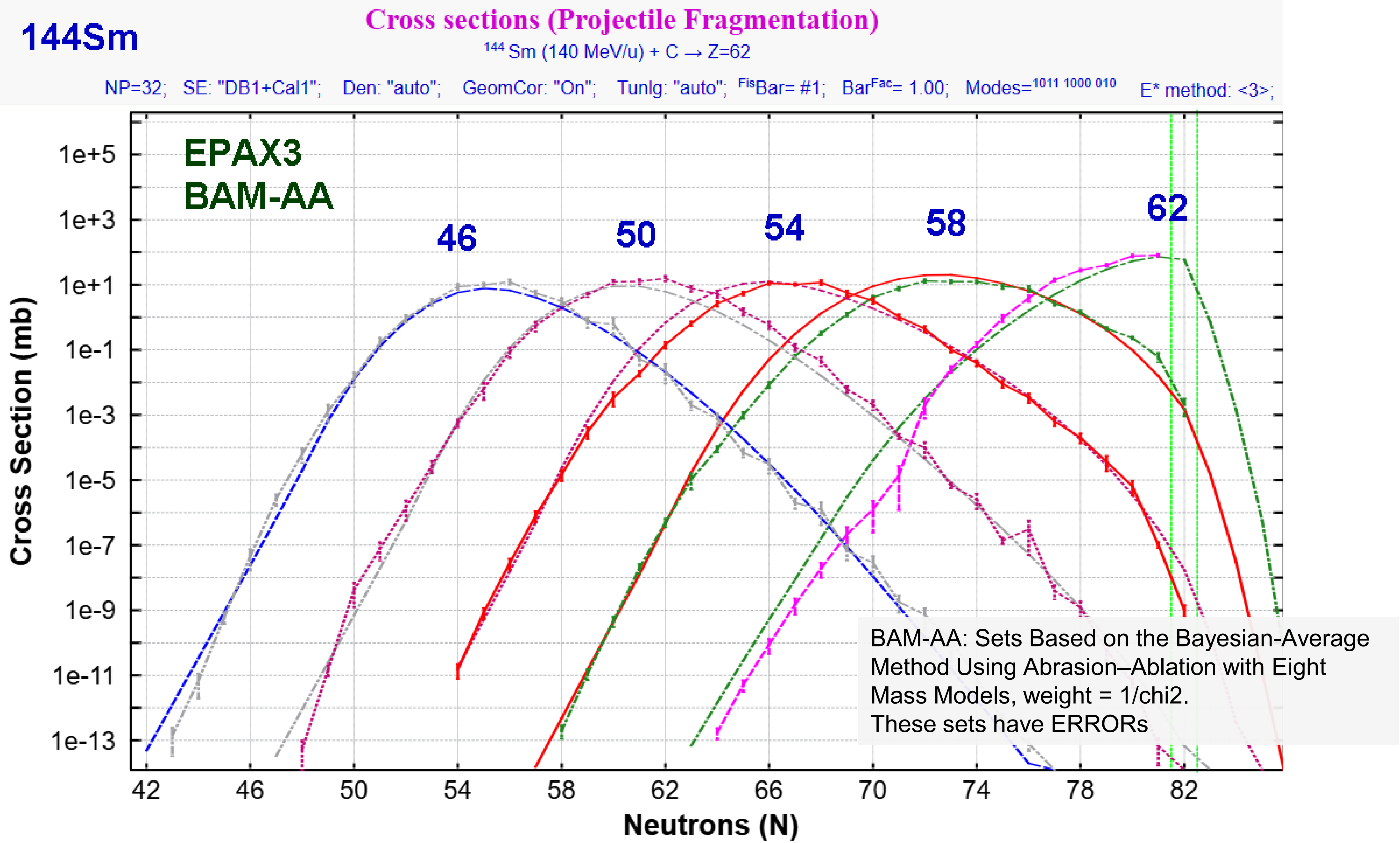
General Bayesian model averaging (BMA): ^{92}Mo → Z-even



General Bayesian model averaging (BMA): $^{144}Sm \rightarrow$ Z-odd



General Bayesian model averaging (BMA): ^{144}Sm → Z-even



General Bayesian model averaging (BMA): ^{78}Kr →

BMA + EPAX3 + experimental CS

Coming soon

General Bayesian model averaging (BMA): ^{124}Xe →

BMA + EPAX3 + experimental CS

Coming soon